

March 2 - Do first Lab together (due in 2 weeks)

Friction

Sliding vs Static  
Molecular level

HW on Friction (assign, work on)  
# 2, 3, 4, 5, 7, 18

March 16 -

Momentum

Rotational Motion

- Angular Momentum  $L$  vs
- Torque
- Rotational Inertia  $I$
- Torque + Momentum

HW on Rotational Motion (assign, work on)  
# 1, 3, 4, 5, 8, 19

March 23 - Talk through 2<sup>nd</sup> Lab (due in 2 weeks)

Fluids start

- density
- pressure
- Hydraulics

(No Homework)

March 30 -

Questions about Lab #2

Fluids

- Buoyancy
- more Fluids start

HW on Statics (assign, work on)

~~6, 7, P18 #1~~

P15 5 7 8 9

P16 12 13

P18 1

P19 1 4

April 6 -

Fluids

- Continuity + Bernoulli
- lots of examples

HW: Fluids - Bernoulli (assign, work on)  
# 1, 6, 7, 8, 20

April 13 -

Thermo Start

- Atoms, molecules, phases of matter
- Heat transfer (conduction, convection, radiation)
- Radiation

Hand Out Midterm (Due next week)

April 20 -

Talk through lab #3

Ideal gas law

Refrigerators

(work on lab 3  
or Next Homework)

April 27 -

Questions on lab #3?

Engines

4 stroke 14 cycle

HW: Thermo

Temperature #1, 8

Refrigerators #1, 2, 3

4 strokes #1, 2, 3

May 4 -

Light - Color

Additive color

sky blue / scattering

HW: P. 31 #2

P. 34 #4, 9, 10

May 11 - Talk through Lab #4 (due in 2 weeks)

Light - Ray optics  
index of ref.  
total internal reflection

(Work on Lab #4 No HW)

May 18 - Quizzes on Lab #4?

Light - lenses  
Horne Sec  
eyes  
ray diagrams

HW : p. 31 # 4, 6 9  
p. 34 # 2 11

May 25 - Modern Physics / cool stuff

No Homework

Final essay due next time  
- work on

June 1 - Modern Physics / cool stuff

## Good Questions

- Inertia,  $F=ma$ , Acceleration  
# 1, 2, 3, 5, 6, 12
- Friction,  $N$ , Impulse, Momentum  
# 2, 3, 4, 5, 7, 18
- Rotational Motion  
# 1, 3, 4, 5, 8, 19
- Static Fluids  
# 6, 7, p. 18 #1
- Fluids - Bernoulli  
# 1, 6, 7, 8, 20
- Thermo HW  
Temperature # 1, 8  
Refrigerators # 1, 2, 3  
4 strokes # 1, 2, 3
- Light - Color HW<sup>21</sup> Light and Refraction # 2  
<sup>34</sup> Light and Refraction # 4, 8, 9, 10
- Light - Ray optics  
p. 31 4, 6, 9  
p. 34 2, 11

# How Things Work S21

Introduction { Mr. Professor Dr.  
Mike Verostek V. } Some combination of these

- Why am I here? My philosophy on learning.
- Who are you?

## Syllabus

- Backstory of the syllabus - it's not all mine! So the course description + learning outcomes are GCC's.
- Basically, my goal is to teach you guys about some of the basic principles that explain how stuff in the world works.
- Text book: interesting, good book, but hopefully everything you need to know will be in class. But obviously if you miss something you can get it from the book.
- Class structure: primarily lecture, but I'll stop a lot to ask questions and talk

Please interrupt me! If things don't make sense, you want clarification, whatever. Raise your hand or just yell.

## Grades and stuff:

- My general attitude is that I hate grades / grading in the usual sense, i.e., give a test, mark wrong or right, put down a number. Class is about learning.
- Unfortunately, GCC sort of requires that I submit grades for everyone at the end of this.
- Basically, if you try to give a good effort, you'll be fine. Do your best and you'll be fine.

Generally your grade will come from 4 places

1) Homework

- Basically depending on how much we get through in class, I'll assign 5-ish questions out of the workbook.

As a general guideline, I'll have those due the following week. If you need a bit more time, that's fine! Just let me know! All in all, the goal will be to have it all done by the end of the semester.

Exception: 2 free assignments!

Participation: just coming to class, asking questions, doing the stuff you're supposed to do. Being respectful of everyone, making the class a good place to be.

This is basically to your benefit. I don't anticipate anyone doing badly on this.

Exams: we'll have 2 "exams"

1) Midterm - will be in place of a homework. Not in class, open notes/book. Will be just like a homework but with questions from each thing we've done.

2) Final exam: Will be a "report" of sorts. It's actually your 5th "lab assignment" - to read about and write about a famous physics experiment.

Labs: Yes there is a lab component! Physics classes almost always have labs, and this is no different. Actually we're basically doing labs just like every school on earth did labs during Covid - paper, pictures, data.

More next week.

(4)

- Don't cheat. Don't just copy solutions. But feel free to work together / talk.

Rough Outline: this might change, but I'll try to hit as much as possible.

If there's stuff you want to talk about, ask!

that's about it, so let's start



Distance  
speed/velocity  
acceleration

{ch. 3}

You probably already know what these mean, or at least have a feeling of what these mean, but we want to make sure we're all on the same page.

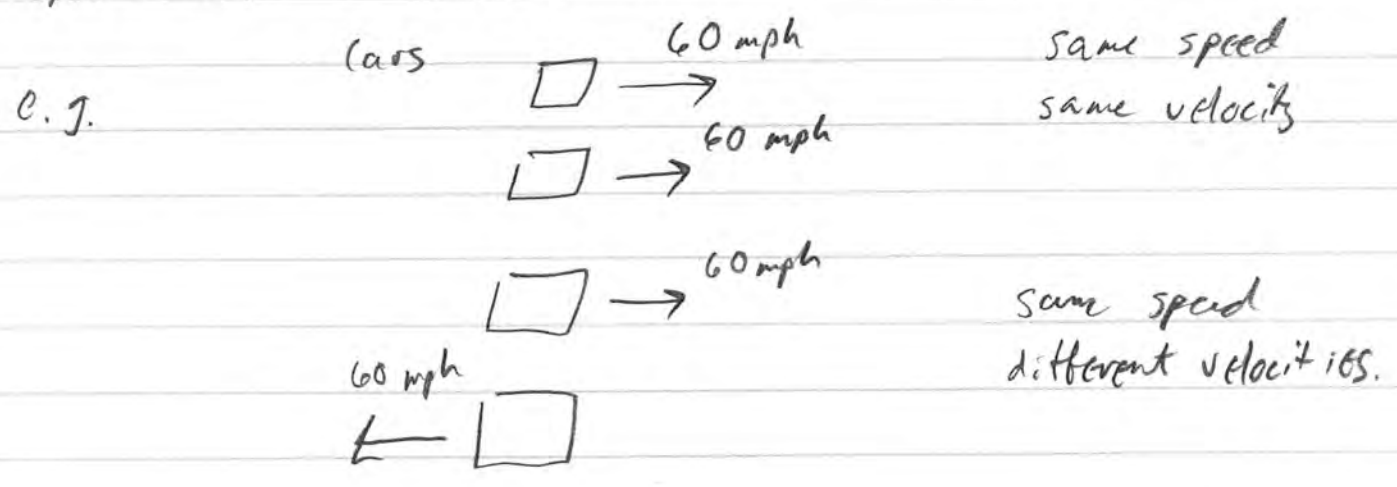
$$\text{speed} = \frac{\text{change in distance}}{\text{time}}$$

How fast distance changes

How fast is something moving? e.g. a car goes 30 miles in 1 hour. Its speed is 30 miles per hour.

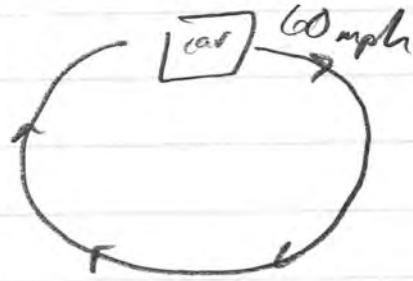
A bicyclist goes 30 meters in 2 seconds. What is their speed? 15 meters per second.

Speed vs velocity? Usually synonyms. And lots of times they are the same. Except velocity is speed with a direction.



(6)

A weird situation:



- what about a car moving in a circle? is its speed changing?
- is its velocity changing?
- don't worry too much about this distinction though

Acceleration: How fast velocity changes, or

acceleration = change in velocity per time

e.g. you hop into your car, you step on the gas and go 0-60 in 5.3 seconds

$$\text{acceleration} = 60/5.3 = 11.3$$

---

ex: ) you jump into the pool.

$\Delta V$  slow down

what if you hit the side and missed the water?

a bigger or smaller? Why?

(7)

$$F = ma$$

ch. 2, 4, 5

m, a

Forces

So the distance / velocity / acceleration stuff isn't that exciting by itself. But I'm going to be saying those words, so it's good to have an idea what they mean when I say them.

## Forces

A long time ago <sup>Isaac</sup> Newton had some new ideas about how things move.

1) All things have inertia

inertia = the tendency to keep going

So an object will keep doing what it's doing unless something else pushes/pulls on it

e.g. you in your seat right now

e.g. this eraser

2)  $F = ma$

F - force

m - mass

a - acceleration

mass = amount of inertia

- How hard is it to change velocity?

acceleration = how fast velocity changes

Cars

Honda Accord

0-60

in 10 sec

Corvette

0-60

in 4 sec

Big  
Truck

0-60

in 10 sec

Accord vs Corvette : compare

Corvette, same  $m$ , bigger  $a$   
→ bigger  $F$

Accord vs Truck : compare

Truck : bigger  $m$ , same  $a$   
still bigger force!

Force has to do with how hard a thing is to move and how much the speed changes

2/23/22

Agenda :

- Questions from last time
- Recap of last week (And Newton's 3<sup>rd</sup>)
- 4 fundamental forces
- Molecules, springs, normal force
- Homework (assign, work on)

Questions from last time:

- 1) Fall grades were sent over last week so they should be arriving shortly.
- 2) REJI folks are planning at least one or two academic advising events soon and will meet with each one of you individually. Not on the calendar yet but it will happen so think of questions / comments.
- 3) The semester ends Memorial Day week (June 1<sup>st</sup> for us)

that's 15 classes (13 after tonight). I'd like to try to finish the week before (May 25), which means getting one double week in there.

Feb 16, 23

Mar 2, 14, 23, 30

Apr 6, 13, 20, 27

May 4, 11, 18, 25

(+1)

Inertia  $F=ma$  accelerations

HW: 1, 2, 3, 5, 6, 12

9

Back to pool example:

You jump into a pool and hit the water. You do it again but this time you hit the step.

Which situation hit you with a higher force? water, step, or neither?

Some examples:

1) Toilet paper: quick vs slow pull: big  $a$ , big  $F$

Big vs small roll: big  $m$ , big  $F$

quick, big more likely to rip

to rip means big force

2) Baseball:

thrown by an MLB pitcher!

you catch a baseball in your bare hands

Does it hurt? Why? Explain in terms of  $F$  and  $a$

Why would a glove help?

### 3) Defenestration:

Anyone know what this means? "The act of throwing someone out a window."

During the fall:

0 - 60 in 3 seconds

(5 stories)

small  $a$ , not too big an  $F$

Land on sidewalk:

60 - 0 in  $1/1000$  s.

big accel, big  $F$ !  
ouch...

---

### Forces, an overview

In the universe, there are 4 fundamental forces

Gravity, Electromagnetic,

$\uparrow$   
most things!

Strong, Weak

Short range

Then there are "derived" forces

e.g. Normal, Friction, spring, tension

## 4 Fundamental Forces

<u>Force</u>	<u>Relative Strength</u>	<u>Force Carrier</u>	<u>range</u>
Gravity	1	Graviton (theorized)	infinite ( $1/r^2$ )
Weak	$10^{-25}$	W and Z Bosons	$10^{-18}$ m.
EM	$10^{-36}$	photon	infinite ( $1/r^2$ )
Strong	$10^{-38}$	Gluons	$10^{-15}$ m.

## Current Theory

Gravity - GR, Einstein

Weak - Electroweak theory

Electromagnetic - Quantum Electrodynamics

Strong - Quantum Chromodynamics



## 1) Gravity

- pulls any two objects
  - Newton's apple
  - Look at the person next to you... About .01% weight of a penny

- Weakest Force : Electromagnetism is billions and billions of times stronger  
e.g. magnet vs Earth

- Weakens with distance  
e.g. astronauts!

## 2) E M Forces

- Springs, normal, tension, friction

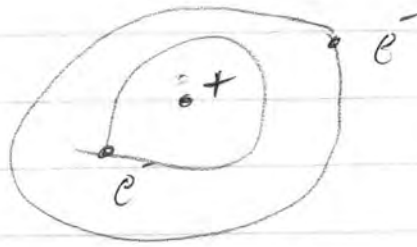
How? Well it's not gravity, and it's not the strong / weak... so there's only one left!

But... how?

Atoms



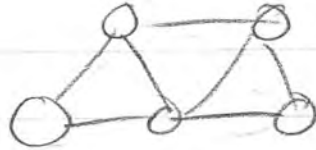
Atoms:



"electrons" orbiting  
around a "proton"

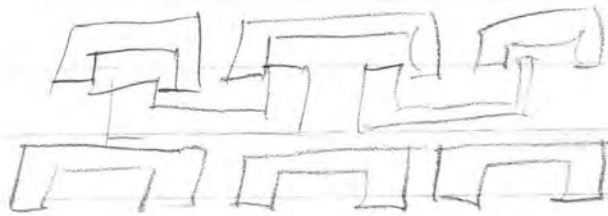
- electrons don't like to be near each other!

Molecules:



some atoms hooked  
together

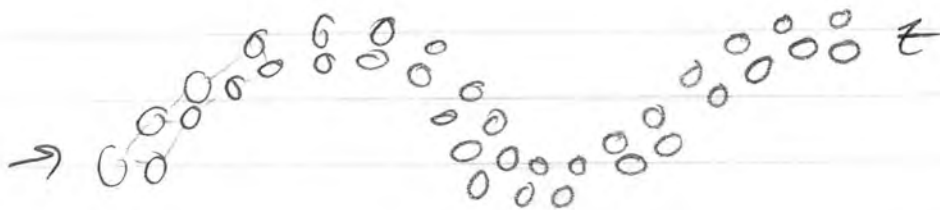
Solids:



Lessos!

solid stuff!

E.g. Springs!



Spring

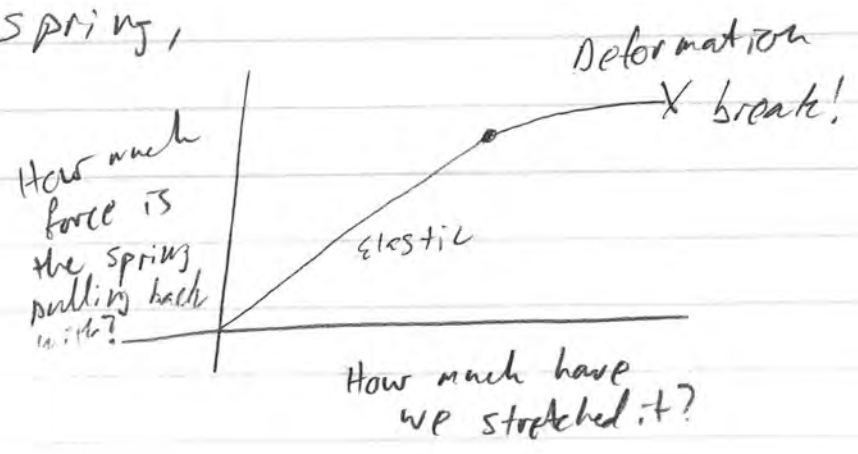
Molecules want to stay where they are!

Compression - push - bonds push out  
Extension - pull - bonds pull back

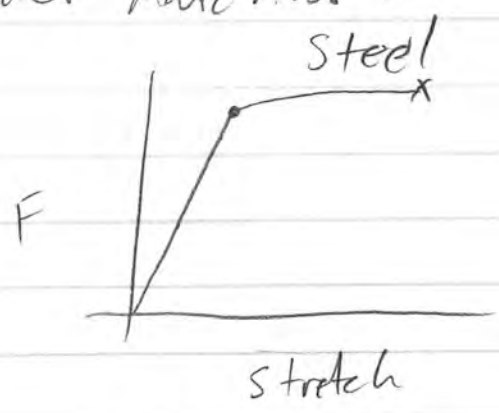
All solids sort of act like this!

More force  $\rightarrow$  Deforms more  $\rightarrow$  More force back

For a spring,



other materials:



Glass?

Brittle - won't stretch much before breaking

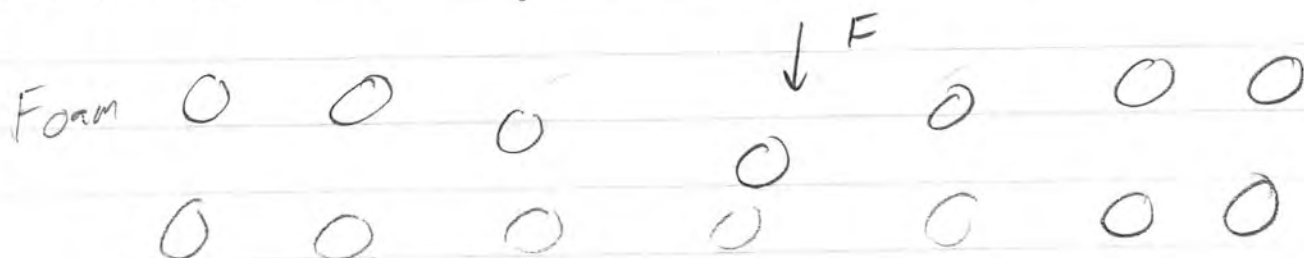
Ductile - will keep stretching for awhile

Would you rather step a bullet with (regular) glass, or steel?

## Normal Force

The normal force is a contact force - for instance when I push on a wall or you sit in a chair.

Surfaces kind of look like this:



- The molecules don't like this! So they push back. Kind of like the spring!
- This happens when you push on any material!
- And is due to EM!

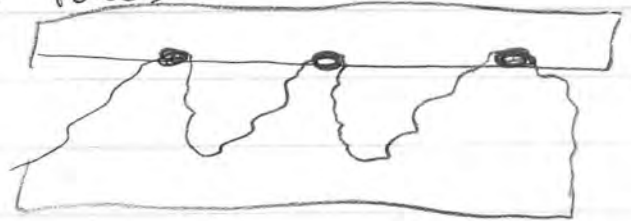
## Friction

- Normal Force is perpendicular to friction
- Friction is proportional to the normal force!

Why? Guesses?

Molecular picture: Electrical forces

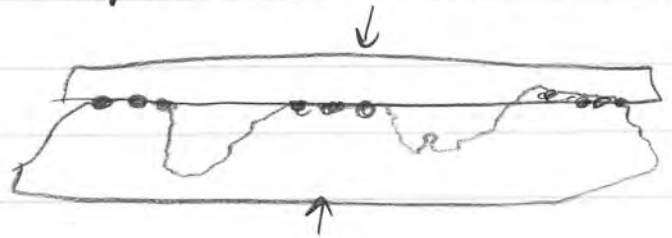
- atoms stick at contact points
- $10^{-4}$  of area



- Friction depends on the number of stuck together atoms!

But how many atoms touch? Depends on Normal force..

More normal force means more atoms touch.

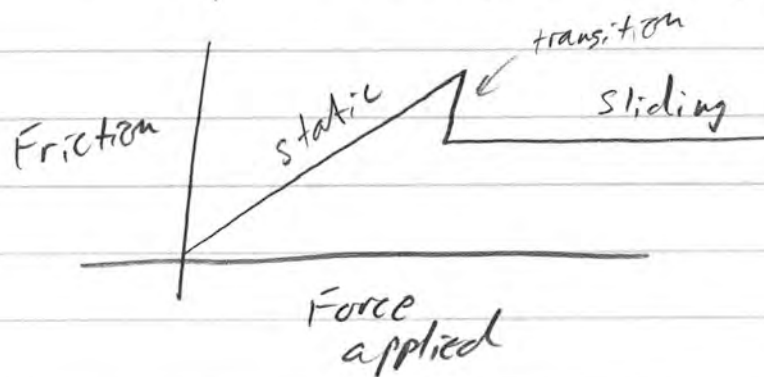


Note: Doesn't depend on surface area of object!

Same # of contact points no matter how big - depends on normal force and kind of material!

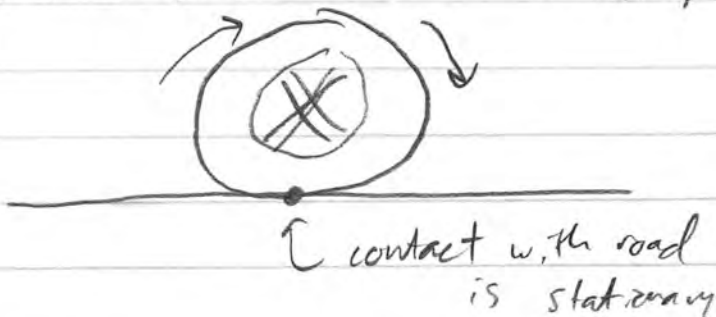
2 types of friction - sliding and static

- Static friction - atoms stick
  - Friction force = applied force
- sliding - atoms "bounce" along, no time to stick
  - always the same, doesn't depend on speed

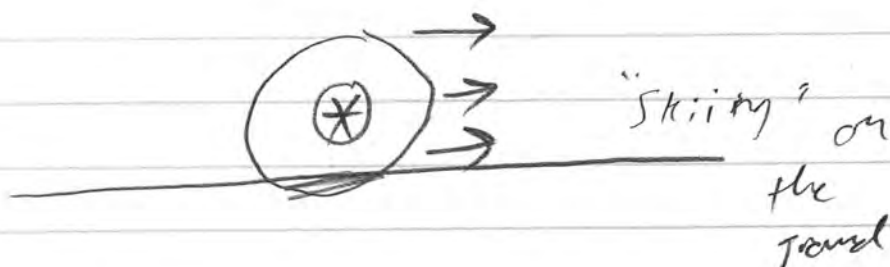


Question: How do cars move?  
 A: Friction!

Bonus: which kind? Mostly static!



VS

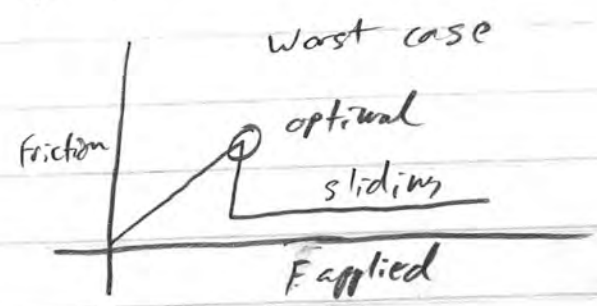
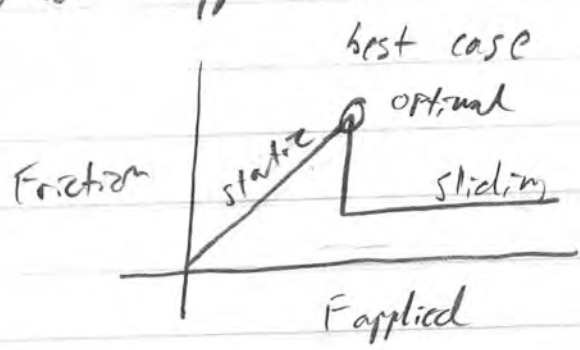


So when you accelerate, what happens?

- Engine applies a force to the wheels to put them in motion.
- Pressing the gas pedal harder makes you accelerate faster... up to a point.

Q: Care to guess what that point is?

What happens to friction graph in ice/rain?

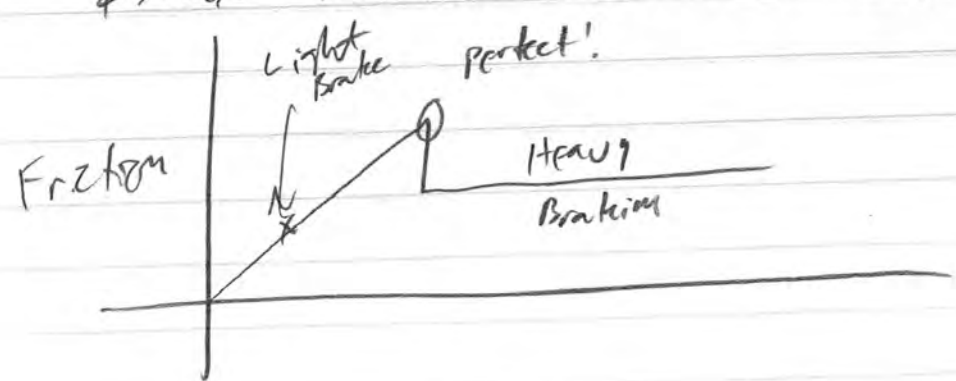


Worst case - harder to accelerate and will easily slide!

• Where is optimal braking force?

- Right at the top of the curve!

Max friction without sliding



- So how do you get that to happen in the Rochester winter?

In the olden days, "pump your brakes"

- shift between sliding and light braking, but how do you know?

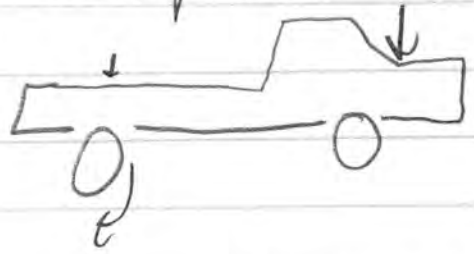
Nowadays, ABS (Anti lock braking)

- ABS automatically knows to brake harder until a slide, then back off, and repeat

Perhaps surprising that braking too hard takes longer to stop!

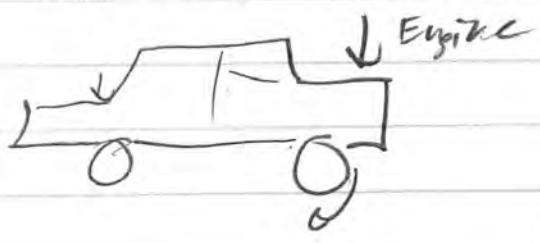
Follow-up question: Front wheel drive or rear wheel drive in the snow?

Pickup with RWD



Rear wheel drive accelerate = poor (Not much weight!)

Car with FWD



Better acceleration  $\frac{1}{2}$  more weight, higher Normal force / friction

Q Why ever have RWD?



Momentum

Angular Momentum

Remember mass: how difficult it is to change motion

Related concept is

momentum: how hard it is to stop

$$\text{momentum} = \text{mass} \times \text{velocity}$$

Why interesting? In an isolated system, total momentum is conserved.

- Floating in space at  $v = 10$  mph
- throw wrench forward
- I slow down

- Floating in space at  $v = 0$
- throw ball right I go left

pool balls  
football players

Similar for spinning things

Inertia → rotational inertia

mass → moment of inertia (rotational mass)

Force → torque

momentum → angular momentum

Rotational inertia = rotating object tends to keep rotating in same direction unless acted on by torque

- ex:
- football spiral
  - frisbee
  - bullet

Amount of Rotational Inertia

- remember, amount of regular inertia related to mass.

Rotational Inertia related to an object's

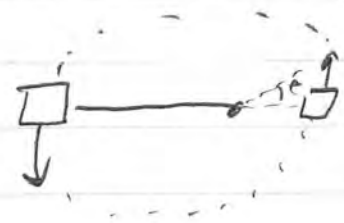
Moment of Inertia  $I = \sum MR^2$

High  $\rightarrow$  Lots of mass spread out far  
Low  $\rightarrow$  Less mass closer to center

Why?

(it angular velocity

$\omega$  constant)  
revolutions per sec



same time to rotate  
 $\rightarrow$

how faster, more rotation



- think weights: closer to middle vs far out which is harder to spin?

Angular Momentum:

- In isolated system must be constant

$m$   $v$   
 $\nwarrow$   $\nearrow$   
mass speed

Rot mass  $\rightarrow$   $I$   $\omega$   $\nwarrow$   $\nearrow$  rot speed

(21)

If in isolated system must be constant

$$I_i \omega_i = I_f \omega_f$$

Q What if  $I_f$  is smaller than  $I_i$ ?

Example: 1) Sit in a spinning chair (preferably with weights)

- start spinning with arms out (High  $I$ , regular  $\omega$ )

then pull arms in ... Low  $I$  so ...  
High  $\omega$ !

2) Figure skaters

- Ever seen a figure skater and they seem to spin really fast, like, forever?

- start with legs out (High  $I$ )

- pull in (Low  $I$ )

Throw in one last concept: Torque

Torque is like rotational force.

Kind of have a gut feeling for high vs low torque ...

Think of shutting a door.

When is it easiest to shut?

- Close to hinge / soft push  
or

- far from hinge / hard push?

Torque = Force  $\times$  Radius

$$\tau = FR$$

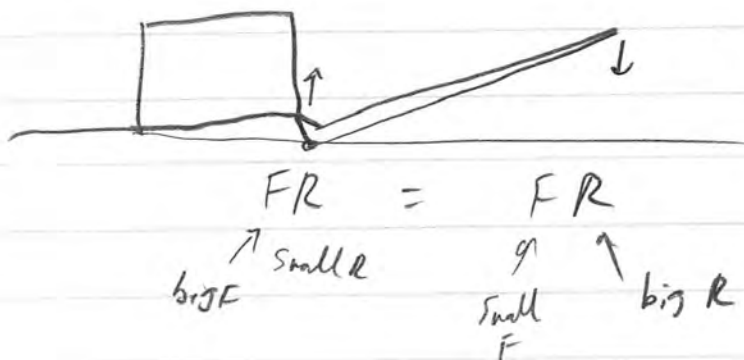
• Force matters more farther from the center!

e.g. wrench



small / big  
wrench?

• Crow bar



Combine these ideas with Angular Momentum

### Example: Top

Why designed like this?



Why mass far out?  $I$   
lots of angular  
momentum

Why small tip? Torque

FR - super small  $R$ , so not  
much torque available to  
stop it

### Example: Frisbees



Throw frisbee, it glides. Has angular  
momentum, so it stays rotating in the same  
spot.

Will it go forever? What stops it (eventually)?

What happens with no spin?

## Fluids

- Both liquids and gases are considered "fluids"
- Fluids are anything that can flow, which liquids and gases both do

Molecules in liquids aren't locked together - they flow around each other.

- Conform to container } think Jellies
- Liquids have approximately fixed volume  
ie they don't really compress

## Density :

- which is heavier, 1 pound of air or 1 pound of lead?
- What's the difference? the volume taken up.

$$\text{Density} = \frac{\text{mass}}{\text{volume}} \quad \sim \frac{\text{weight}}{\text{volume}}$$

## Pressure

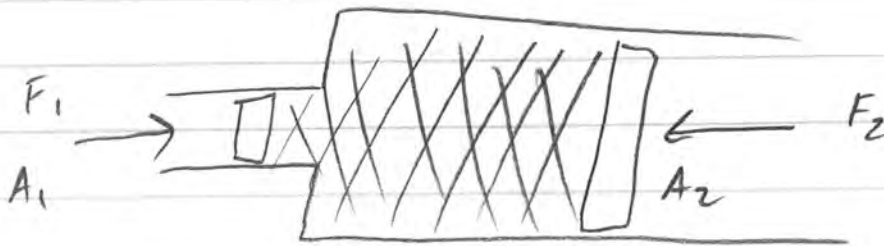
$$\text{pressure} = \frac{\text{Force}}{\text{Area}}$$

I weigh ~ 190 lbs. I apply 190 lbs of force when I stand. But it hurts more if I step on your hand with stilettos than my shoe.

# Pressure example: Hydraulics

- Snarshoes / Stiletto :  $F = \text{const}$ , Pressure  $\sim 1/\text{Area}$

Inside a fluid  $P = \text{constant}$  at some depth



$$P = F/A$$

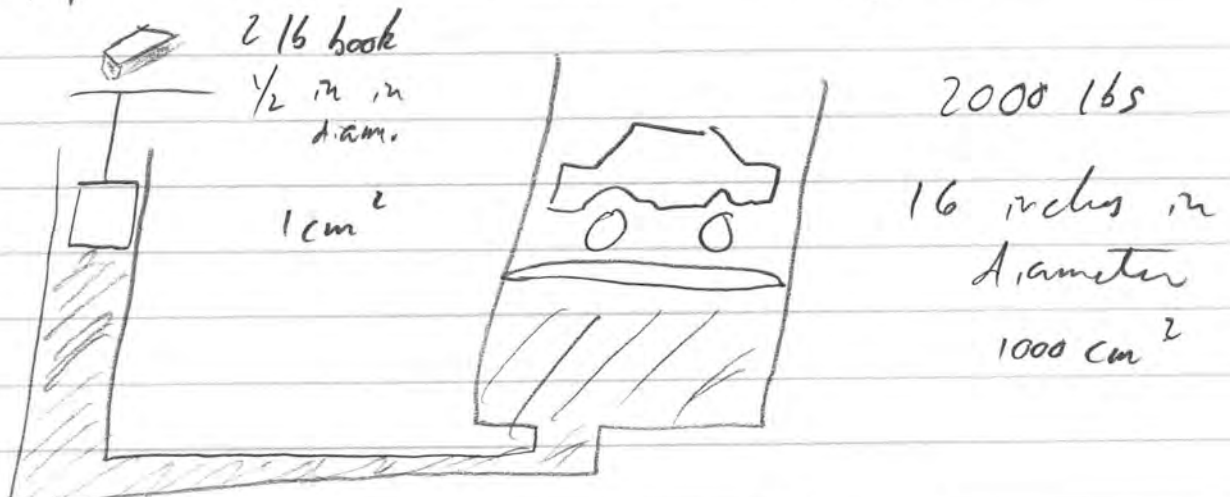
$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = \left( \frac{A_2}{A_1} \right) F_1$$

$\propto$  multiplying factor!

$A_2 \gg A_1$ , then a small  $F_1$  makes a big  $F_2$ !

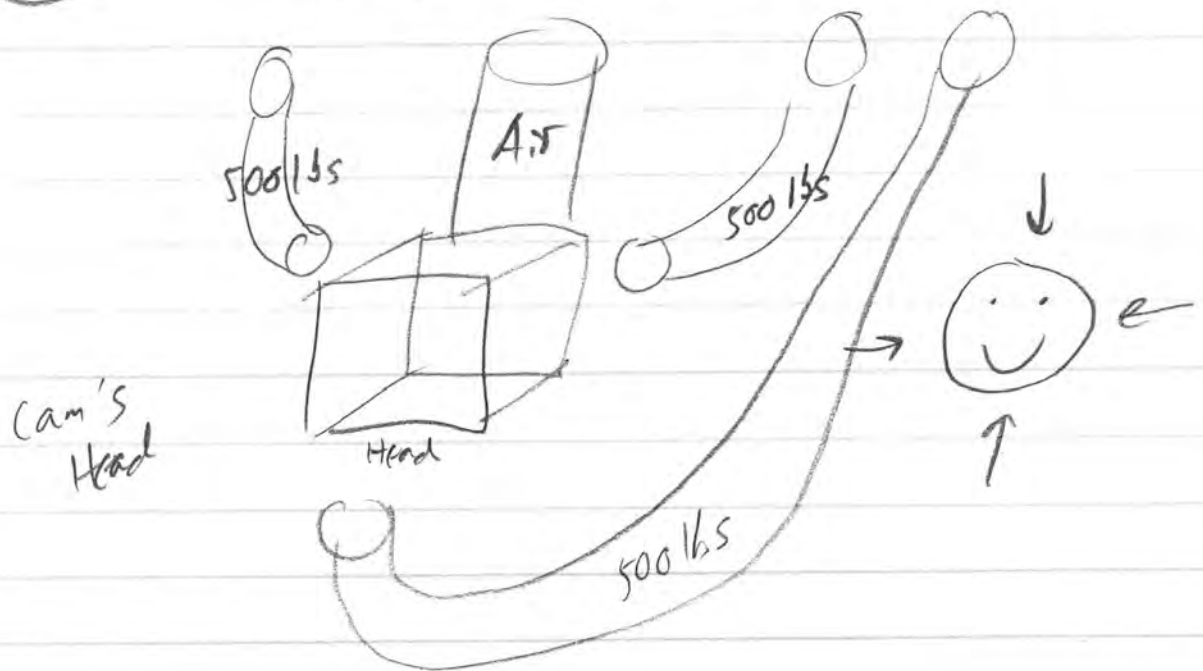


But you don't get it for free... What's the limitation? Volume = constant

Bed of Nails : guesses as to why you don't get skewered if you stand on a bed of nails?

Snow Shoes : Large area, same force (weight), so each bit of snow holds less up.

Air pressure : What is air pressure? Guesses?



But Cam is pressurized - the pressure inside pushes out  
pressure out pushes in  
all is OK.

Counter examples? When pressure in  $\neq$  out?

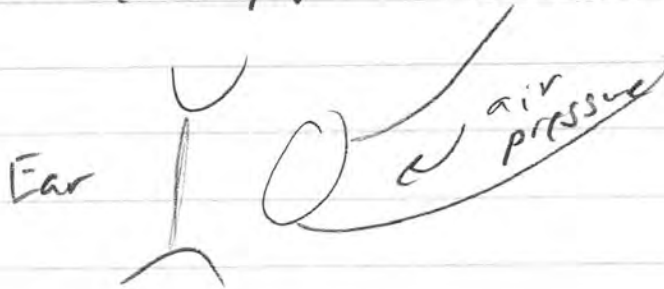


Air pressure is big! - Water glass description  
try it!

Straws

Some consequences of air pressure:

When do your ears pop? More or less air above ear



Pressure only depends on depth

Water Tower - why so tall?



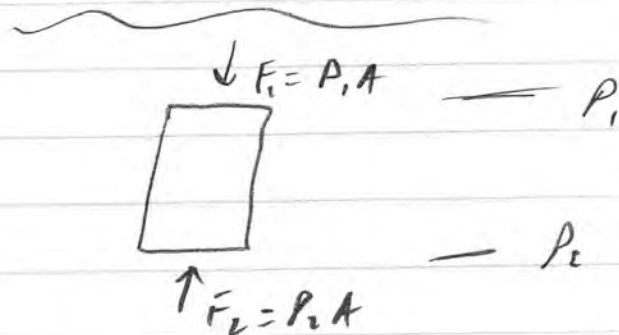
↑ pressurizes pipe - water wants to go through pipe /  
up into houses. But too much demand = pump  
can't pressurize. Height provides the pressure.

## Buoyancy

Pascal's idea:

$$F_{\text{buoyant}} = F_2 - F_1$$

$$= P_2 A - P_1 A$$



$P_2 > P_1$ , so  $F_2 > F_1 \rightarrow \text{Floating!}$

But remember, gravity is always pulling down.

So the net force on an object in water is

$$F_{\text{net}} = F_{\text{buoyant}} - F_g$$

If weight  $>$  buoyant force, the thing sinks.

What if block doubled in width? Length? Height?

$F_{\text{buoyant}}$  <sup>only</sup> depends on volume

not shape!

Even weird shapes!

Archimedes - Trick to find out  $F_B$   
Special case of object submerged in water  
The weight of 1 gal of water is 8 lbs

(29)

$$F_{\text{net}} = F_B - F_g \rightarrow F_B - F_g = 0$$
$$F_B = \text{weight of 1 gal}$$

In fact,

$F_B =$  the weight of the equivalent volume of fluid displaced

Again,  $F_B$  only depends on volume underwater

E.g. put 1 gal can underwater  
 $F_B = \text{weight of 1 gal of water}$   
 $= 8 \text{ lbs}$

Put 2 gal can under,  
 $F_B = \text{weight of 2 gal of water}$   
 $= 16 \text{ lbs}$

Doesn't matter what's in can!

1 gal of air	$F_B = 8 \text{ lbs}$	(float)
1 gal of lead	$F_B = 8 \text{ lbs}$	(sink)
1 gal of water	$F_B = 8 \text{ lbs}$	(float - stationary)

Know  $F_B \rightarrow$  know  $F_{net}$

1 gal lead  $F_{net} = 81bs - 901bs$  sink!

1 gal ice  $F_{net} = 81bs - 71bs$  float!

Will it float? (compare  $F_B$  vs weight)

$$\text{dens. } F = \frac{\text{weight}}{\text{volume}}$$

more dense than water  $\rightarrow$  sink

less dense than water  $\rightarrow$  float

Why does a battleship float?

- Heavy but also has lots of air, less dense.

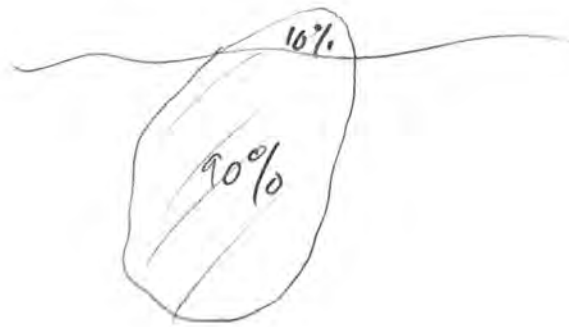
- Weight of volume displaced (HUGE!)  
Bigger than weight of boat

Doesn't have to be water - works for any fluid

density of helium < density of air  
 $\rightarrow$  helium balloon floats

density of mercury > density of iron!  
 $\rightarrow$  iron floats in mercury

Icebergs - ice density is about 90% of water  
so it only takes 90% the volume of water to  
equal the same weight as a block of ice  
→ icebergs have about 90% of their volume  
underwater



---

## Moving Fluids

Ideal Fluids - idealized picture

viscosity: water vs Honey  
friction, lose energy

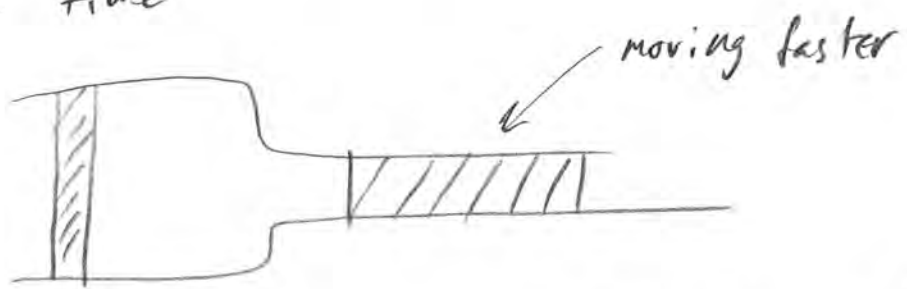
compressibility: air kind of springs  
e.g. basketball

ideal = non viscous incompressible

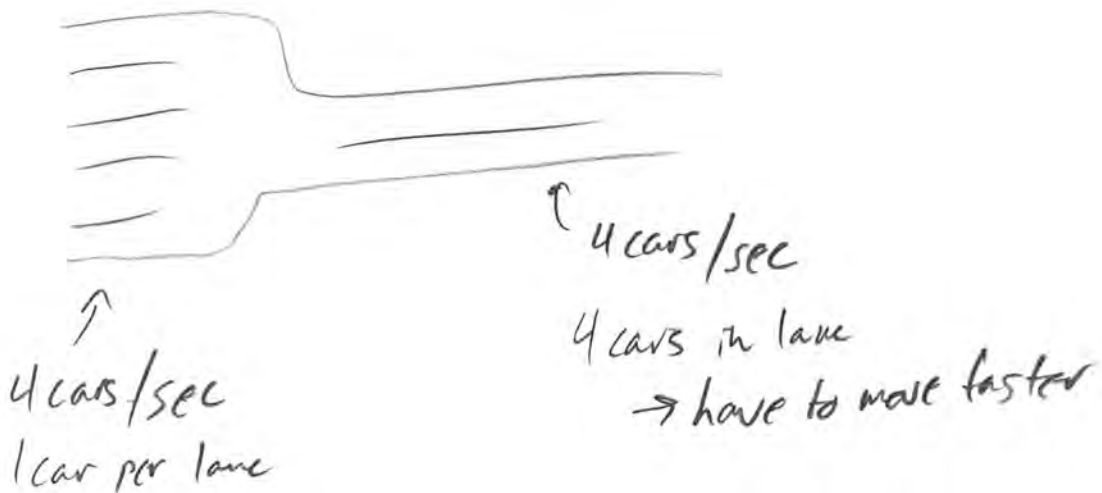
# Continuity

32

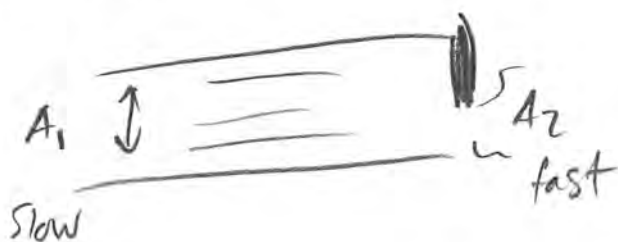
Same amount of fluid passed a given point in same time



Otherwise there would be a "buildup" of fluid



Garden Hose :



Bernoulli : fast fluid has less pressure

(33)

I can prove it... piece of paper, blow across the top.

Or from continuity,

• in smaller area,

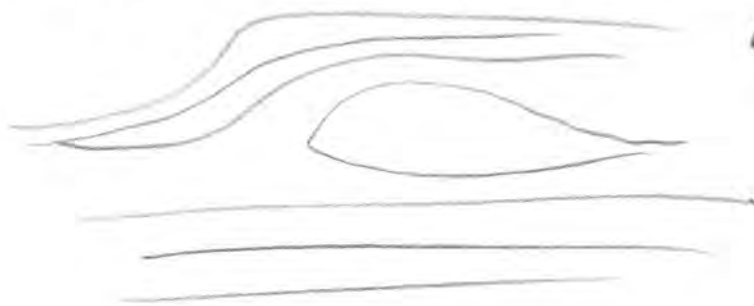
↳ faster

↳ had to have accelerated

↳  $P_1 > P_2$  (carburetors)

Model of air flow = "Laminar Flow"

- Airplane wing



Less area,  
less pressure,  
more l.f.t.

Baseball, Frisbee



Magnus  
Force

# Thermo Topic Overview

34

- Thermal Energy
  - Molecules/Atoms
  - Heat / temperature
  - Phases of matter
    - Solid Liquid Gas
    - Melt Boil Sublimate
    - Molecular Model
    - $\Delta$  Energy at same temp
    - Evaporative cooling
  - Heat Capacity

- Heat Transfer
  - Conduction, Convection, Radiation

- Black body Radiation
  - Incandescent, Heat seeking, Night vision, Red Hot
  - Greenhouse

- Thermal Expansion
  - Gas, Solid

$$PV = NkT$$

Refrigerators

Engines



## Atoms and Molecules

- Atoms, ~100 elements  
Building blocks, can't be divided
  - Molecules, billions of combinations e.g. water molecule
- For us, the smallest chunk of a substance

## Phases of matter:

Solid - Fixed positions  
vibrating

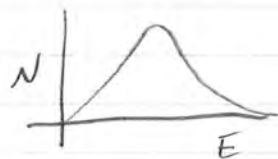
Liquid - touch, slide past (telly bears)  
vibrations + rotations

Gas - not touching  
velocity + rotation

Add energy to solid  $\rightarrow$  more vibrations  $\rightarrow$  break bonds  
 $\rightarrow$  more vibration/rotation  $\rightarrow$  break bonds

Heat vs Temperature: Temperature =  $\frac{\text{Average Kinetic Energy}}{\text{molecule}}$

Average some faster, some slower



Temperature doesn't include binding energy

Energy = Kinetic + Binding Energy  
 $\uparrow$   $\uparrow$   
 temperature phase of matter

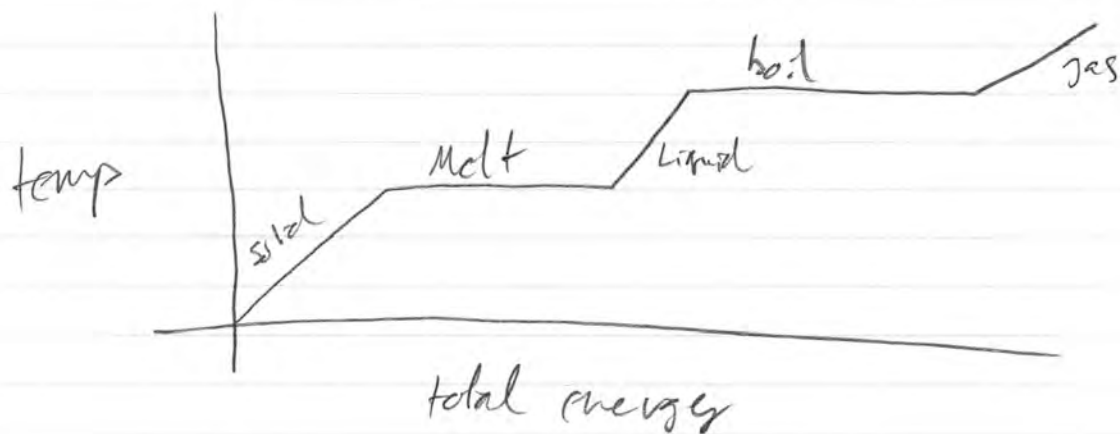
Add heat to solid  $\rightarrow$  vibrate faster temp  $\uparrow$

Melt  $\rightarrow$  Break bonds temp constant

Add heat to liquid  $\rightarrow$  vibrate faster temp  $\uparrow$

Boiling  $\rightarrow$  break bonds temp constant

Add heat to gas  $\rightarrow$  move faster temp  $\uparrow$



Higher temp - glass of water with little ice or glass of ice with little water?

Non intuitive - kinetic energy determines how often molecules are able to break off the ice/become water or vice versa

Heat Capacity - How much heat to increase temperature by  $1^\circ\text{C}$

big heat capacity  $\rightarrow$  lots of energy to raise temp a little bit

Heat capacity = mass  $\times$  specific heat

↑  
more energy  
to heat  
bigger object

↑  
material - can change  
with variety of factors  
water = 4    ice = 2  
wood = 1.7    Alum = .9

Heat is expensive!!

Energy to boil 5 cups of water = E to bring  
small car 0-60

Evaporative cooling - e.g. glass of water

- Fastest pop off  
if fastest pop off, what  
happens to avg KE?  
Decreases  $\rightarrow$  temp decreases

- blow on hot soup vs cover hot  
coffee  
- sweat

How is heat transferred?

Conduction, convection, radiation

• Conduction: move thermal energy through material

- hot poker: molecules at end vibrating really  
fast, hit others, they vibrate faster,  
etc. down the poker

Conduction examples: pot cooking, cold tile floor,  
Hot coffee / paper cup

Convection: kind of like moving a hot object to a new place

fluid (air, water) comes in contact with an object with different temperature, then becomes more/less dense, moves

air near a candle gets warm, expands, moves upward  
cold air takes its place, warms, etc.

- Forced air heat, atmosphere/climate patterns

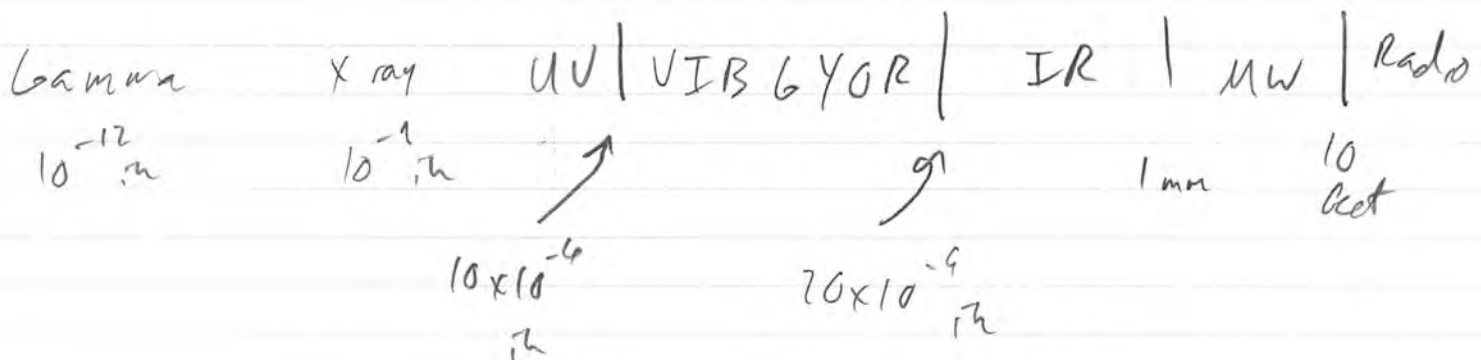
↳ why air registers on floor?  
Air conditioning?

Smoke rises? Not really - hot air floats!

Radiation recall Maxwell!

What is EM radiation? Light

EM spectrum



- Light carries energy!
- Doesn't require air!
- ex - sun, fire

All things emit light!!

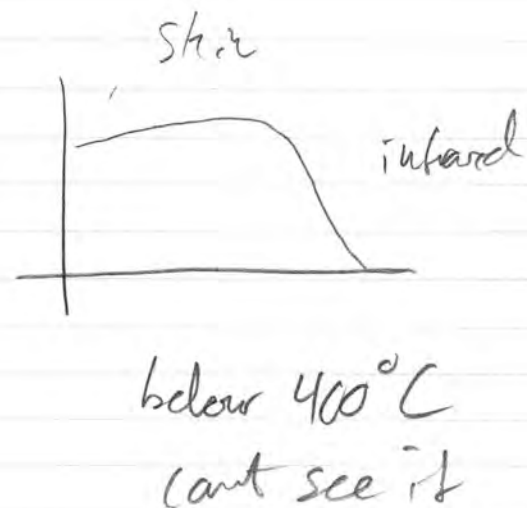
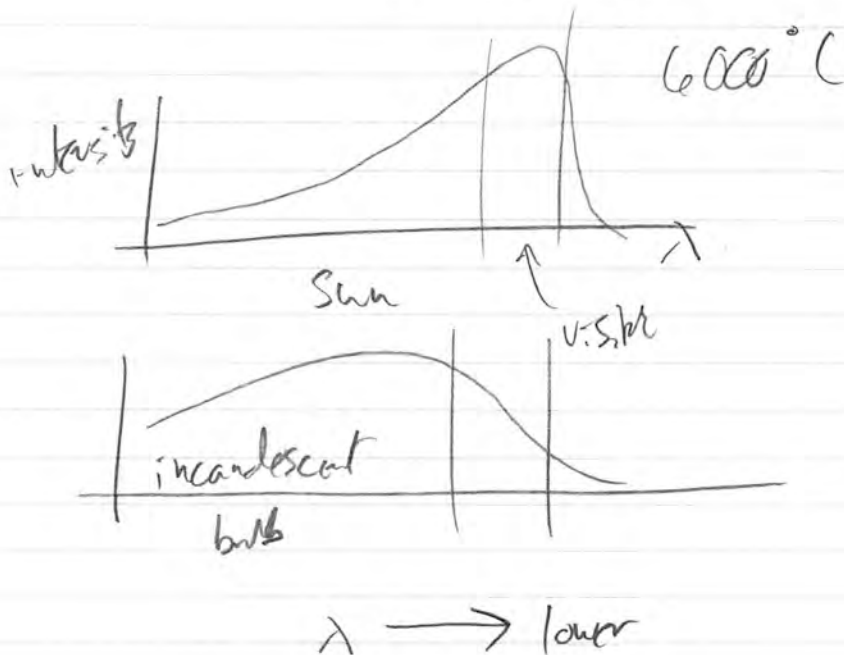
All things with a temperature give off light

- "Blackbody Radiation" is the thermal EM radiation emitted

Emitted light depends on temperature

- Hotter object gives off shorter  $\lambda$  "bluer" light
- Hotter gives off more light

Heat transfer: Hot object  $\rightarrow$  give off light  $\rightarrow$  light travels  
 $\rightarrow$  hits object  $\rightarrow$  light absorbed



Emissivity - fraction of light absorbed/emitted

$\epsilon = 1$  black perfect absorber/emitter

$\epsilon = 0$  white/mirror absorb/emits nothing

- Roof color, hwy roof
- snow melts on blacktop
- black clothes on hot day

Greenhouse Effect

# Ideal Gas Law

(41)

- 3 processes for  $V \downarrow$
- Adiabatic Process

## Refrigerator

- Basic, real (freon)
- Heat pump

## Car Engines

In Lab:

Solid thermal expansion - relatively straightforward!

Gas thermal expansion - More complicated!

Governed by ideal gas law:

$$PV = NkT$$

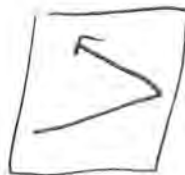
Annotations:

- $P$ : Pressure (arrow pointing up)
- $V$ : Volume (arrow pointing up)
- $N$ : # particles (arrow pointing up)
- $k$ : Boltzmann constant (arrow pointing up)
- $T$ : temperature (arrow pointing up)
- Sometimes  $PV = nRT$

Why?

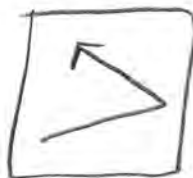
$P$  has to do with force - how often are particles hitting the sides of the box?

1)  $P$  vs  $N$   
( $T, V$  constant)



2x pressure,  
2x particles

2)  $P$  vs Vol  
 $P \propto \frac{1}{Vol}$



$\frac{1}{2}$  volume  $\rightarrow$  2x pressure!

3)  $P$  vs  $T$

(42)

remember  $\text{temp} \sim KE \sim v^2$

So  $2\times$  velocity =  $4\times$  Temp

Pressure at  
 $2\times$  velocity:



$2\times$   
bounces

but also  $2\times$  harder!

$P \propto T$

So  $2\times$  velocity  $\rightarrow 4\times$  pressure

$$P = k_B N T \frac{1}{\text{vol}}$$

$$PV = k_B N T$$

So how does gas expand/contract (i.e., change volume)

1)  $P = \text{constant}$

Temp  $\uparrow$ , volume  $\uparrow$

Particles go faster, walls push out



2) Temperature Constant  $\rightarrow PV = \text{constant}$  (43)

$V$  decreases, pressure increases

Squeezing particles at same speed

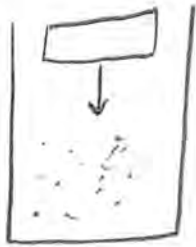
3) Adiabatic Process

"No energy in or out"

"Fast"

$$V = NR \frac{T}{P}$$

$$V \downarrow \rightarrow T \uparrow P \uparrow \uparrow$$



Piston hits atoms, speeds them up,  $\rightarrow$  hotter!

Hotter + smaller  $\rightarrow$  Lots o' pressure!

Likewise,

$$V \uparrow \rightarrow T \downarrow P \downarrow \downarrow$$

Particles bang against side, give up energy to move side, slow down

cooler + bigger  $\rightarrow$  Little bit o' pressure

Take home message:

(44)

Gas: Expand fast  $\rightarrow$  gets cold  
Compress fast  $\rightarrow$  gets hot

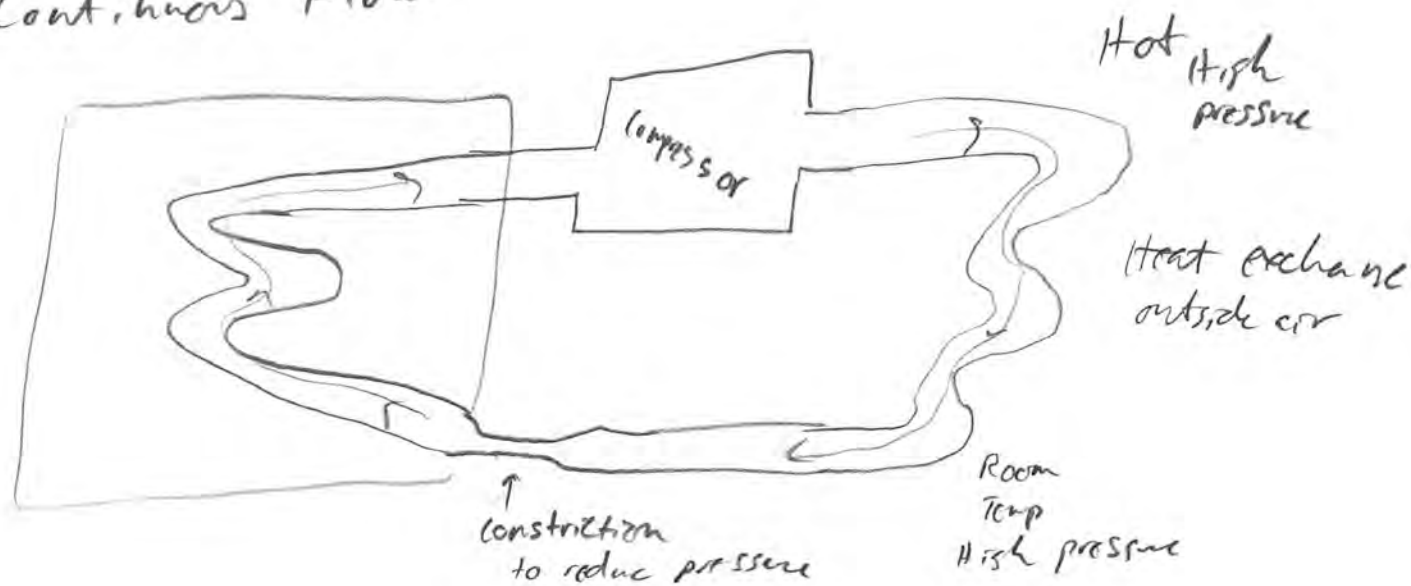
## Refrigerators:

Basic - Compress outside - gets hot (Piston)  
wait, gives off some energy,

- move inside
- Expand - gets cold  
wait, absorbs energy
- move outside

Taking energy in the refrigerator and pumping it outside

## Continuous Flow



Efficiency;

(45)

Heat exchanger: Fan why?

Block why?

Remove dust why?

Defrost why?

Tightly sealed fridge door why?

Household fridge "Freon" CFC (chloro fluoro carbon)

Same process but amplified

Freon High pressure = Liquid

Low pressure = gas

Binding energy!

Compress, gets hot ← gives off E

and becomes Liquid & TURNS off E to do

gas → liquid

Expansion

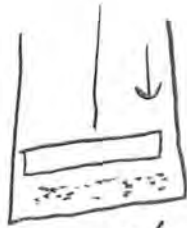
gets cold

becomes gas ← takes in E

# Engines

(46)

Imagine a piston with fuel in it



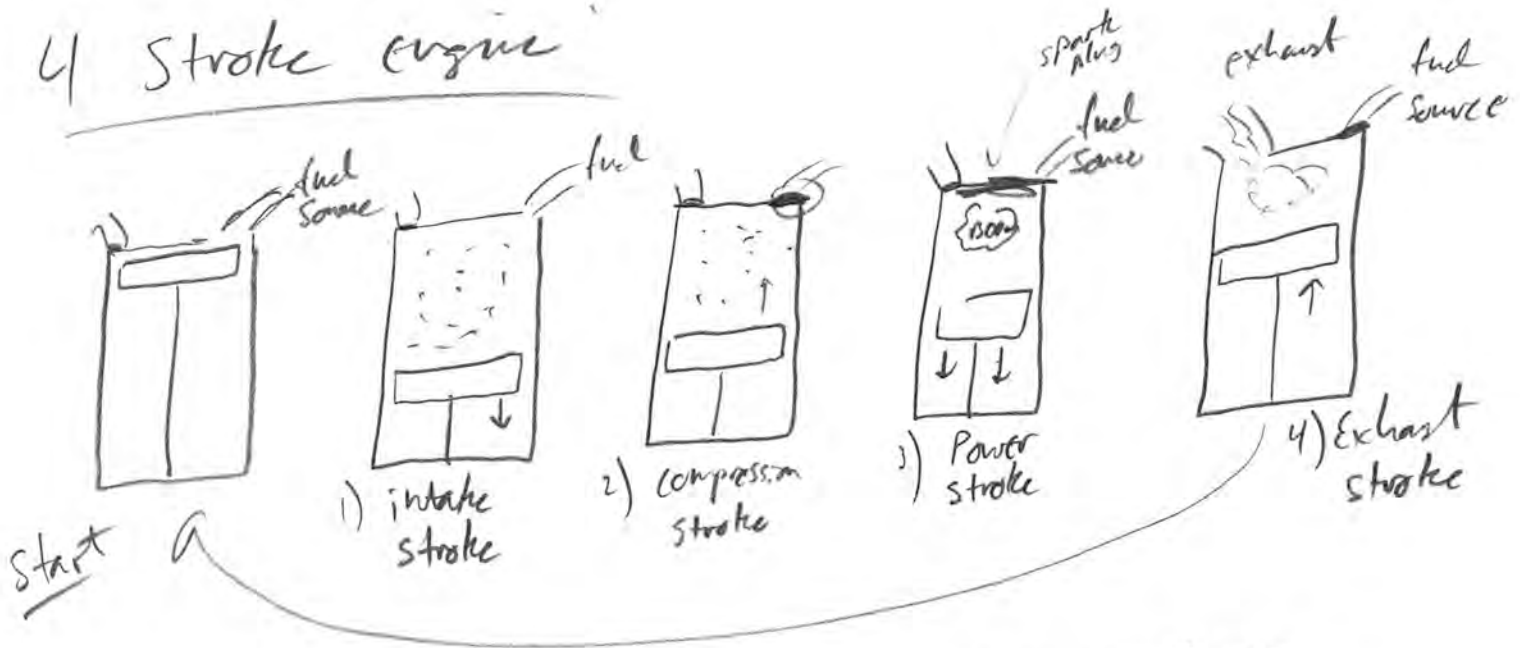
compress fast  
→ gets hot!



fast!  
explode with  
spark

Basic idea of engine.

## 4 Stroke engine

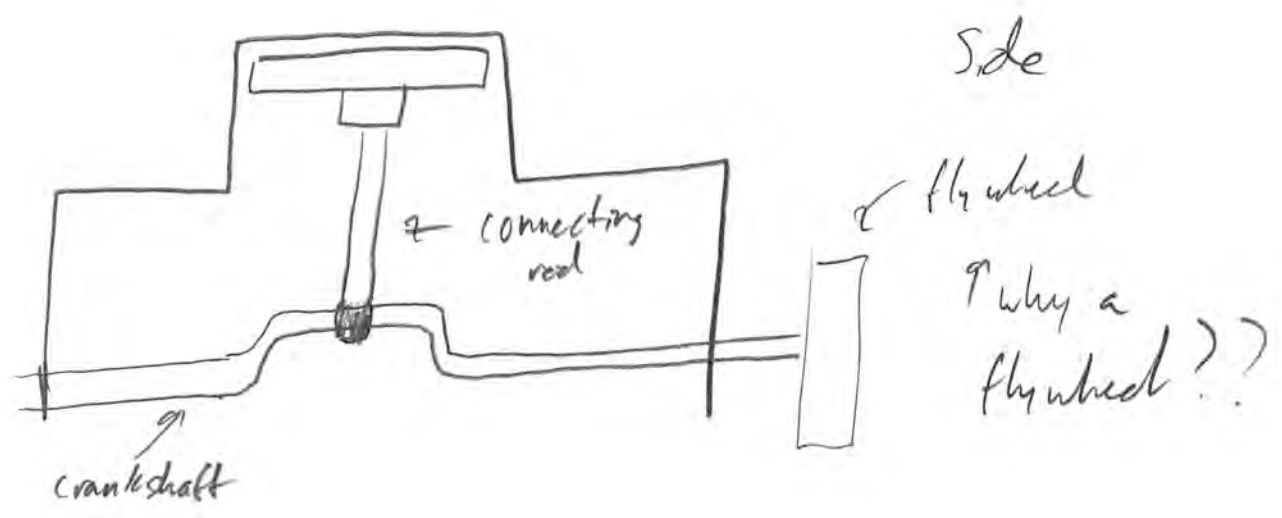
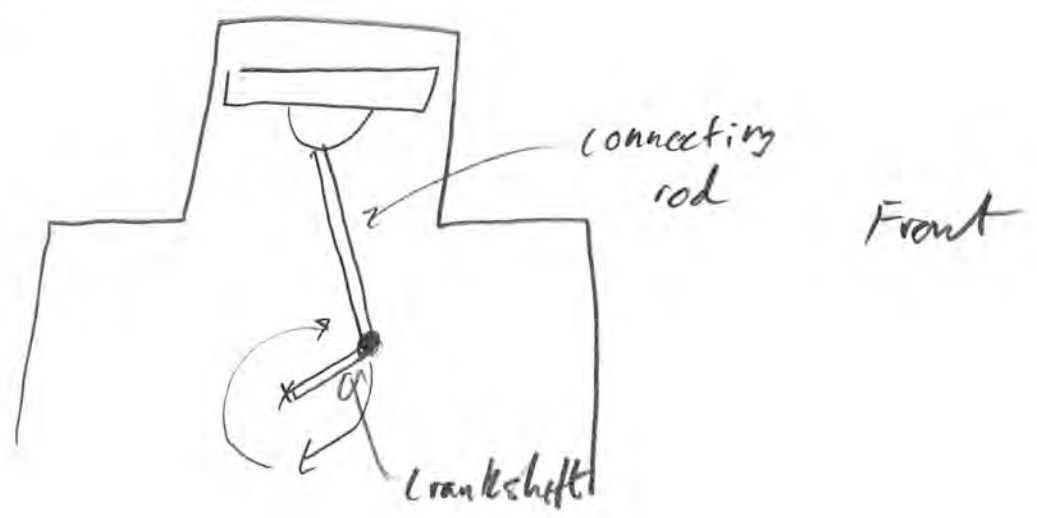


This is how car engines work at a cartoon level. - Why Adiabatic?

- What are some design decisions that need to be worked out here?
- How to transfer power, how much fuel, how to open/close holes, what kind of fuel ... among others...

Transferring power - pistons move up and down but  
tires rotate... How to fix?

# Crankshaft

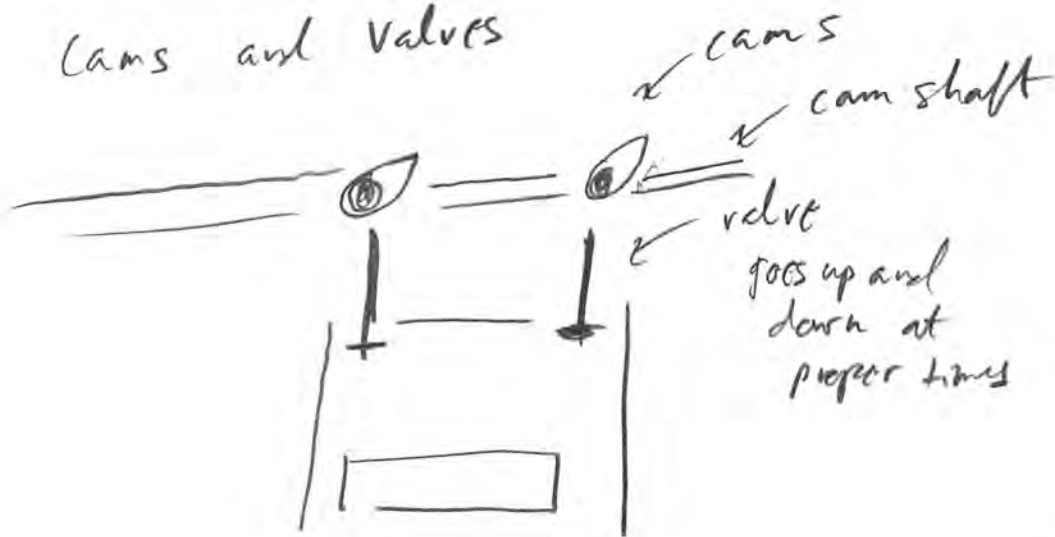


One problem solved!

48

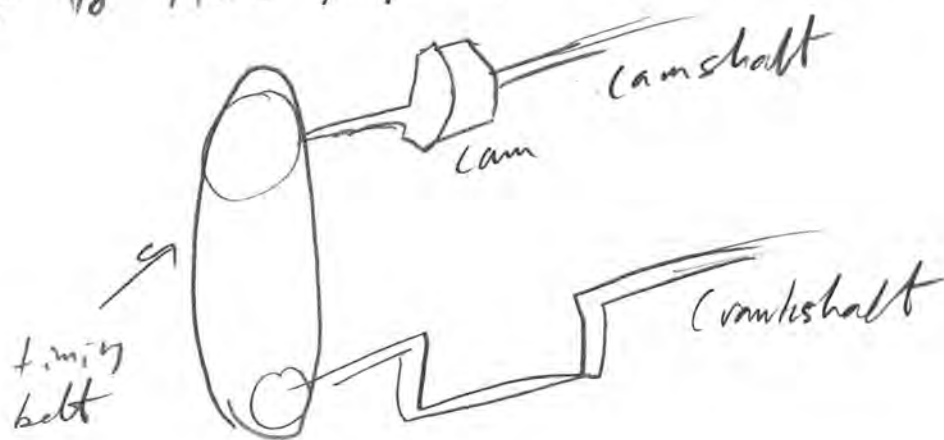
How to let gas in?

Cams and Valves



Cams on cam shaft press down on valves to open them.

How to time properly? Connect to crankshaft



Alignment determines when cams will open  
make sure timing belt is on correctly!

## "Variable Valve timing"

(49)

- Lets more fuel in by extending amount of time it's open on intake stroke when more power is needed

- Octanes - higher octane ignites at higher temperature
  - need ignition to happen at correct time, so putting wrong octane in your car can cause "knocking"

- How much fuel?

old: carburetor

Still on tractors / mowers



Faster a.r.  
pulls more  
fuel (lower  
pressure!)

new: fuel injectors

spray fuel directly in piston

Volume / Force :

(50)

Power  $F = PA$  Big  $A \rightarrow$  more force

$L$  stroke length Longer time  $\rightarrow$  more inertia

$$\text{Volume} = A \times L$$

C.g. 4.0 litre engine

Volume from Top dead center to Bottom Dead center

---

Turbo - push more air into cylinder  
burn more gas

used exhaust to do so

---



# Light!

(51)

## Waves

all waves are

similar

- sound
- ocean
- light



Wavelength

Wavelength vs object size  
determines effects

Light  $\sim \frac{1}{\text{million}}$  of feet

sound  $\sim 3$  feet

radio FM  $\sim 15$  feet

AM  $\sim 1$  mile

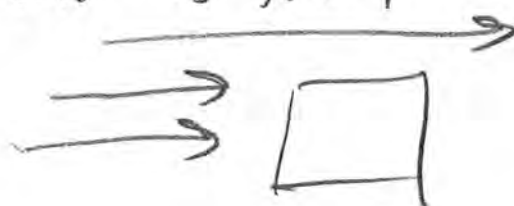
ocean  $\sim 300$  feet

## 3 cases

a)

wavelength much smaller  
than object

light vs shadow  
water vs jetty



rays

b)

Wavelength much bigger than object  
doesn't notice it's there!  
doesn't "see" it

(52)

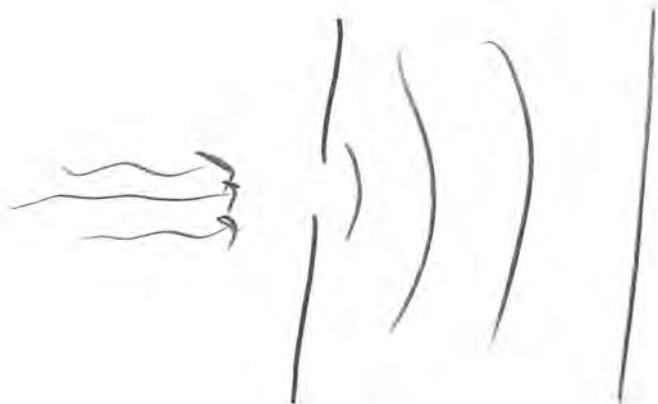
Water + dock post  
Sound + pencil

Microwave door vs Light  
Hole Microwave

c) wavelength  $\approx$  object

Weird!

Spreads out



blue  $\sim 10 \times 10^{-6}$  shorter  
red  $\sim 20 \times 10^{-6}$  longer

(53)

Why is the sky blue?

Molecules  $\angle$  Blue light

mostly ignored

Molecules  $\angle \angle$  Red light

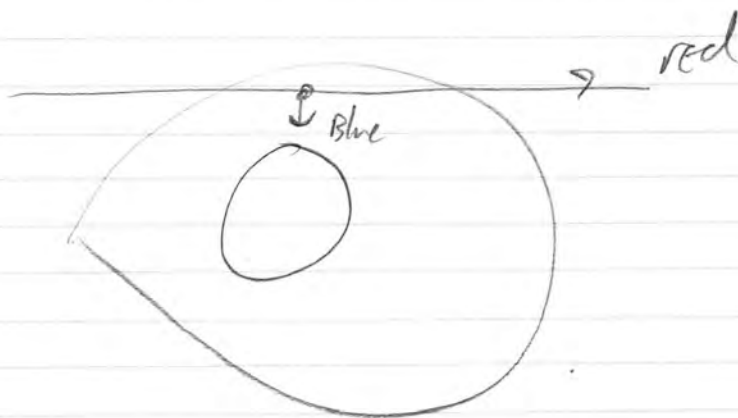
even more ignored

Water droplets? Light  $\rightarrow$  clouds!  
reflects in all directions

Blue light scatters more than red



red sunsets - Lots o' scattering! No Blue left!



AM - 100s of miles around mountains  
 FM - blocked by hills / buildings, Local

Wavelength  $\ll$  Object

Ray optics

Index of refraction - speed of light slower in materials

$$c = \frac{3 \times 10^8 \text{ m/s}}{n}$$

$$n \geq 1$$

$$n_{\text{air}} = 1.0003$$

$$n_{\text{glass}} = 1.3 - 1.6$$

$$n_{\text{diamond}} = 2.6$$

$n$  determines refraction and reflection

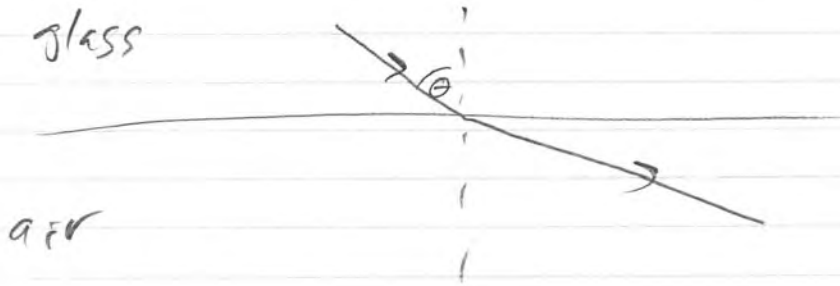


- 1) Bend Away from Fast material
  - 2) Bend least bend on
  - 3) Blue bends more than red
- sand / powder are white

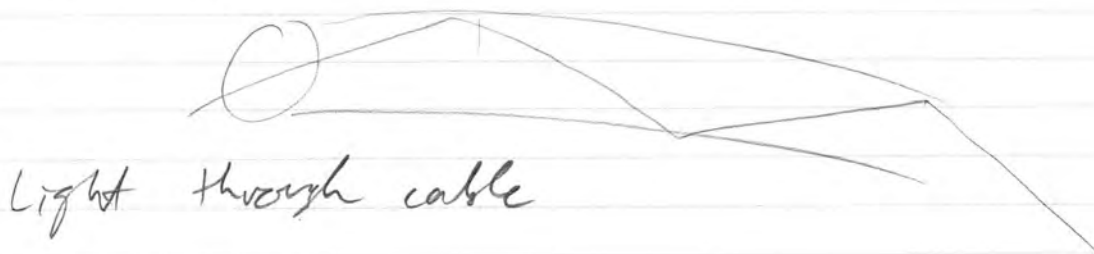
reflection - mirror 90%  
 glass 4%

# Lab #4 - Snell's law discussion

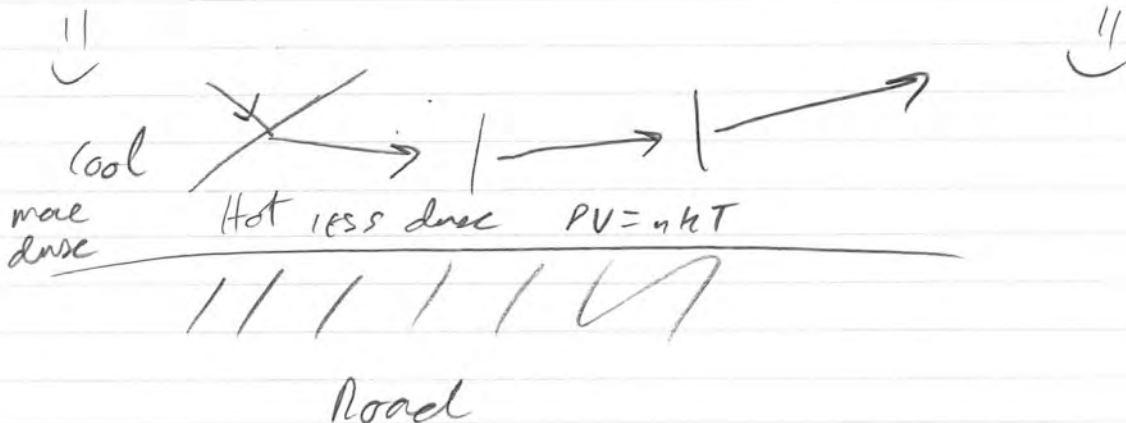
Interesting case:



What if  $\theta$  gets bigger? Total internal Reflection  
- Fiber opt. 25



Road Mirage:



A small slice of Einstein Gravity

How Things Work

Instructor: Mike Verostek

Spring 2022

## 1632: Galileo

We could probably start a discussion of Einstein's Gravity, also known as General Relativity, in Ancient Greece with some of the folks there who were thinking about geometry, math and philosophy. But Galileo's observations really set up the contrast between classical ways of thinking about space and time and how Einstein thought about them.

In 1632, Galileo wrote in his *Dialogue Concerning the Two Chief World Systems*, the following passage:

Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though doubtless when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still. In jumping, you will pass on the floor the same spaces as before, nor will you make larger jumps toward the stern than toward the prow even though the ship is moving quite rapidly, despite the fact that during the time that you are in the air the floor you will be going in a direction opposite to your jump. In throwing something to your companion, you will need no more force to get it to him whether he is in the direction of the bow or the stern, with yourself situated opposite. The droplets will fall as before into the vessel beneath without dropping toward the stern, although while the drops are in the air the ship runs many spans. The fish in their water will swim toward the front of their bowl with no more effort than toward the back, and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air. And if smoke is made by burning some incense, it will be seen going up in the form of a little cloud, remaining still and moving no more toward one side than the other. The cause of all of these correspondences of effects is the

fact that the ship's motion is common to all the things contained in it, and to the air also. That is why I said you should be below decks; for if this took place above in the open air, which would not follow the course of the ship, more or less noticeable differences would be seen in some of the effects noted.

How observant! It's this kind of insight that set the whole story of relativity in motion. And although these kinds of things might seem sort of obvious today, they weren't way back in 1632. For Galileo, he was occupied with the relative movement of things (e.g. the floor and you jumping).

What he asserted was basically this: if you're in a car going 60 mph and you throw a ball forward going 5 mph, you (the person traveling along with the car) will see it going 5 mph. However, your friend sitting on the side of the road will see it going with the speed of the car plus the speed that you threw it with, 65 mph. It must be going faster! See Figure 1.

## 1687: Newton

Around 1687 Isaac Newton figured out that the force of gravity between two objects (really two masses, see Figure 2) is given by,

$$F_G = \frac{Gm_1m_2}{r^2}, \quad (1)$$

where the  $m_1$  and  $m_2$  are the two object masses,  $r$  is the distance between them, and  $G$  is the *gravitational constant*. Sometimes people just call this "Big G," and it's one of the fundamental constants of nature. Big G is equal to  $6.67 \times 10^{-11} \text{ N/kg}^2\text{m}^2$ .

Newton's law of gravitation basically says that the force of gravity between two objects is strongest when the masses are big and close together. Newton's gravity predicts a ton of stuff *incredibly* well! We figured out how pretty much all the planets and stars move with it. But from the beginning Newton knew something was

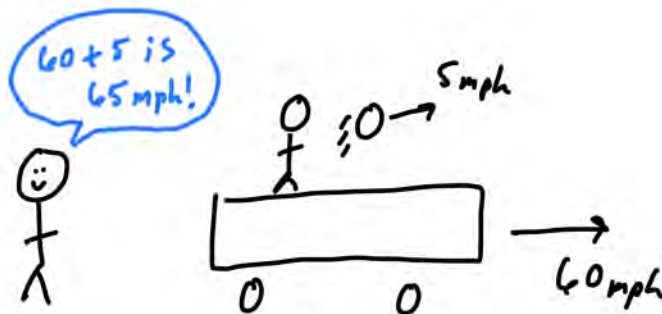


Figure 1





Figure 2

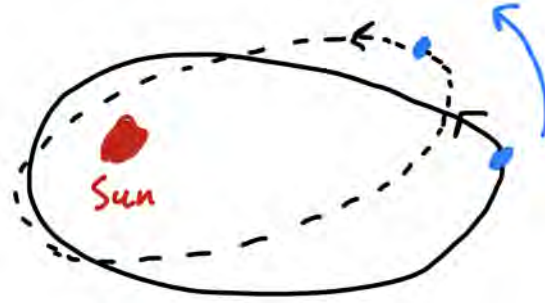


Figure 3

wrong with it, or at least that he hadn't explained everything. He said, "It is inconceivable that inanimate matter should, without the mediation of something else, which is not material, operate upon and affect other matter without mutual contact."

What Newton is saying is that he doesn't understand how the gravitational force *works* at a fundamental level. His equation describes things well, but he doesn't know why. Imagine this: you have two bowling balls near each other. Newton's law of gravitation says that there is a gravitational attraction between them. You go and slap a blob of clay onto one of the bowling balls. The force of gravity between the two bowling balls got bigger. But how did the one of the balls know that you took the clay, moved it, and stuck it to the other ball? Newton didn't know.

As years went by we started to figure out more things were wrong with Newton's law of gravitation, perhaps the most famous of which was Mercury's Perihelion Shift (see Figure 3). Basically, when planets orbit the sun, they do so in ellipses (kind of flattened circles). Perihelion is the point when the planet is closest to the sun. It turns out that when planets are orbiting the sun, the point at which they reach perihelion slowly rotates. For the most part, Newton predicts this (in fact, this is essentially how we discovered Neptune; we saw that Uranus was orbiting in a way that needed another planet nearby for its orbit to make sense, and that planet was Neptune!).

The reason for this is Mercury is being pulled on by the sun, but also by all the other planets. These smaller pulls cause the perihelion rotation. However, astronomers eventually realized that no matter how detailed their calculations, they couldn't get the prediction for Mercury's rate of perihelion shift quite right. We accounted for everything, but we were still about 43 arc seconds (.012 degrees) of movement off per

year (yes that's a very small amount, but people have been staring at the sky for centuries so our data was super accurate).

## 1862: Maxwell

Fast forward a couple hundred years, and James Clerk Maxwell pretty much solves all of electromagnetism (without modern mathematics by the way!!!). He figured out that all electric and magnetic fields are governed by just four equations:

$$\begin{aligned}\vec{\nabla} \cdot \vec{E} &= 4\pi\rho & \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} &= -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} & \vec{\nabla} \times \vec{B} &= \frac{1}{c} (4\pi\vec{J} + \frac{\partial \vec{E}}{\partial t})\end{aligned}\tag{2}$$

Yes this looks complicated but they're not too bad! First, the  $\partial$  symbol just means "a little change in" something." So  $\partial E$  means "a little change in the electric field." Then,  $\vec{\nabla} \cdot$  means "divergence of." Divergence has to do with how much a field spreads out. This is in contrast to  $\vec{\nabla} \times$  which is the "curl" and has to do with how much a field... well... curls. If you know how much a field spreads out and curls, you know all there is to know about it.

But don't worry about all that. What's important is that the top left equation means that electric fields ( $E$ ) are made by charges (that's what the  $\rho$  is - electric charge). The top right one means that magnets always have a north and south pole. If you cut a magnet in half, it just becomes another smaller magnet. You can't isolate the north or south pole!

The bottom left and bottom right are a little more complicated looking, but their interpretation is actually straightforward. The bottom left means that changing magnetic fields ( $B$ ) produce electric fields ( $E$ ), and the bottom right one means that changing electric fields produce magnetic fields. So if nothing else is around, a changing  $E$ -field will start to generate a  $B$ -field, but then that  $B$ -field will start to generate an  $E$ -field, which will generate a  $B$ -field, and so on and so forth. This alternation is an *electromagnetic wave*, AKA light!!!

Maxwell's equations determine that the speed of this wave (the speed of light!) is equal to

$$c = 3 \times 10^8 \text{ m/s},\tag{3}$$

or about one foot per nanosecond. This is the universal speed limit. Nothing travels faster than light.

But wait... Say you're traveling along in a car going 60 mph with a camera. You take a picture and the flash goes off (see Figure 4). You see the light traveling at the speed

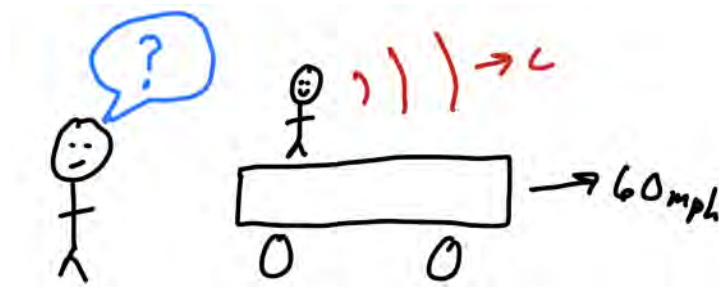


Figure 4

of light  $c$ . But what does your friend standing outside the car see? Basic logic, and all of physics up until the early 20th century, assumed that your friend would see the light going at  $c + 60$  mph. Except that doesn't agree with Maxwell!!! His equations say that the speed of light is  $c$ , full stop. Period. End of story. So which is it? Does your friend see the light going at  $c$  or  $c + 60$  mph?

This set up a major showdown in the history of physics, Galileo vs Maxwell.

## A (Somewhat) Brief Digression on Geometry

I'll leave you with that cliffhanger for a moment while we do a brief digression on geometry, which will be useful in a minute.

Think way back to middle school geometry class for a moment. Chances are at some point you saw the relation shown in Figure 5, called the Pythagorean Theorem. The Pythagorean Theorem says that for a right triangle (a triangle where one of the angles is 90 degrees), the sides are related by the equation

$$a^2 + b^2 = c^2. \quad (4)$$

I bet you never thought you'd ever see that again did you!?! So what does the Pythagorean theorem really tell us? It's really a measure of distance. It's saying that there are 2 ways of expressing the distance between the bottom left and top right

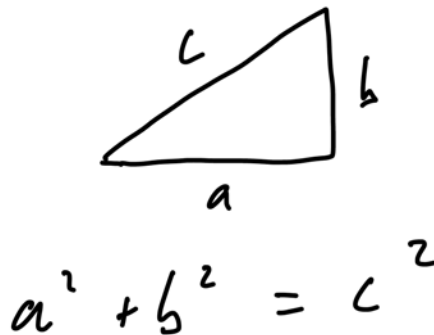


Figure 5

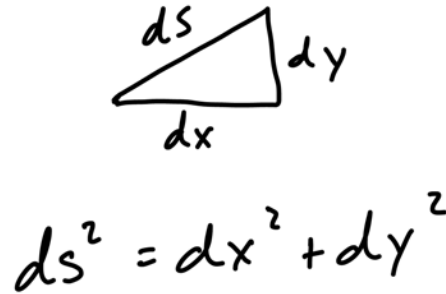


Figure 6

points of the triangle (the points connected by the hypotenuse labeled  $c$ ). On one hand the distance is just  $c$ , but Pythagoras tells us that we can write  $c$  as the square root of  $a^2 + b^2$ . Keep that in mind as we move forward.

What we're going to do now is write the Pythagorean Theorem that you know so well in a very fancy way. So as we move along and weird Greek letters and crazy symbols show up, just remember that at the heart of it all is just good ol' Pythagoras.

First, the Pythagorean theorem doesn't say anything about how big  $a$ ,  $b$ , and  $c$  are. They could be huge, or they could be really, really small. Remember a couple sections ago I said that the symbol  $\partial$  means "a small change." In math, the letter  $d$  also means "a small change" (maybe that's unsurprising, the two look super similar). So if I drew a really, really, infinitesimally small right triangle (as in Figure 6), I could write

$$ds^2 = dx^2 + dy^2. \quad (5)$$

In fact, this relationship scales up to three dimensions too, as shown in Figure 7. In 2 dimensions, we wrote the distance between two points,  $ds$ , as  $ds^2 = dx^2 + dy^2$ . So it's only natural that in 3 dimensions we can write the distance between two points,  $ds$ , as

$$ds^2 = dx^2 + dy^2 + dz^2. \quad (6)$$

However, in a sense, we arbitrarily labeled  $x$ ,  $y$ , and  $z$ . In fact, there are lots of ways to specify where things are. Consider Figure 8, which shows an arrow coming from the origin (the point where the  $x$  and  $y$  axes cross). How can we label the point where the arrow ends? Well we could say, just like in the Pythagorean theorem, that it's a certain distance along the  $x$ -axis then a certain distance up the  $y$ -axis. But we could also label the end of the arrow using the coordinates  $r$  and  $\theta$  (the Greek letter theta), which are written in blue.  $r$  would describe the length of the arrow, while  $\theta$  describes how far the arrow is pivoted above the  $x$ -axis. Either set of coordinates gets us to the same spot at the end of the arrow. And in fact we can switch back and forth between the two labels via the relationship  $x = r \cos \theta$  and  $y = r \sin \theta$  (these are just the definitions of sine and cosine, but if you're not familiar with those, don't worry about it!).

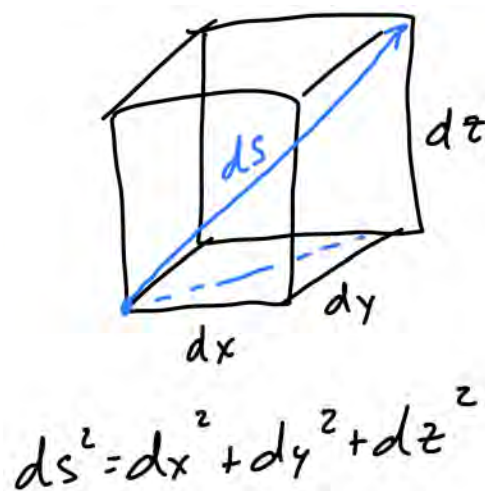


Figure 7

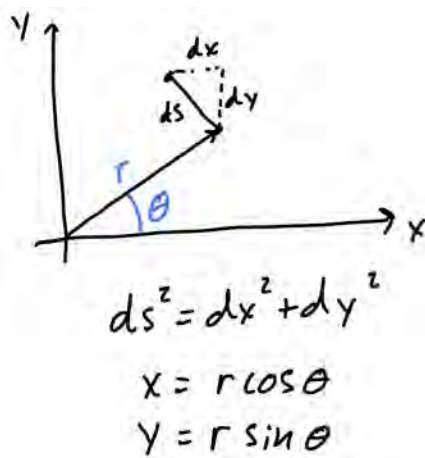


Figure 8

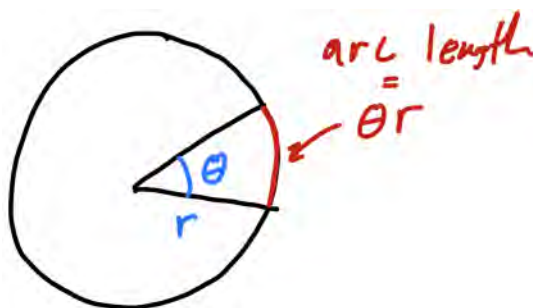


Figure 9



Figure 10

Since we can write the location of the point  $(x, y)$  with the coordinates  $(r, \theta)$ , we should also be able to write a small change in distance using both coordinates. We already know that  $ds^2 = dx^2 + dy^2$ . But what about  $r$  and  $\theta$ ? Well, to traverse the distance  $ds$  shown in Figure 10, we need to move out a little bit in the  $r$  direction,  $dr$  (shown by one of the dotted lines). Then we need to connect back over to the other end of the  $ds$  hypotenuse (shown by the other dotted line). How far is that distance? At some point you might have learned that the length of an arc swept out by an angle  $\theta$  is given by  $r\theta$  (shown in Figure 9). So for a really small angle, like the one in Figure 10, the length of the arc swept out is  $r d\theta$ . That is the length of the other leg of the triangle formed by the dotted lines, with the hypotenuse being  $ds$  and the third leg being  $dr$ .

Thus, just like we could write  $ds^2 = dx^2 + dy^2$ , we could also write the distance  $ds$  as

$$ds^2 = dr^2 + r^2 d\theta^2. \quad (7)$$

So even though  $ds$  is the same, the way we measured it is different. Indeed, this whole discussion is a complicated way of saying that we can measure  $ds$  in a bunch of ways, but no matter which one we use, we will always measure  $ds$  as having the same length. This makes sense! In fact, it kind of has to be this way!!! It would be weird if the length of  $ds$  changed. That would be kind of like the distance between your house and the house down the street being different depending on the route you take there. You went a different way, but the length of the street doesn't change!

Now I'm going to write these expressions for  $ds$  in a fancy way that seems needlessly complicated, but is actually quite useful! First, remember that anything multiplied by zero is zero, and anything multiplied by one is itself. So I could write,

$$ds^2 = dx^2 + dy^2 = 1(dx^2) + 0(dxdy) + 0(dydx) + 1(dy^2) \quad (8)$$

All I did there was put a 1 in front of the parts of the Pythagorean Theorem and a

zero in front of two other terms that are multiplying  $dx$  and  $dy$  together. Now, I'm going to introduce something called a *matrix*. A matrix is just a collection of numbers inside some brackets:

$$g_{\mu\nu} = \begin{bmatrix} g_{00} & g_{01} \\ g_{10} & g_{11} \end{bmatrix} \quad (9)$$

So what does this mean? Well first off,  $g_{\mu\nu}$  is just the name of the matrix (the thing on the right side of the equals sign).  $\mu$  is the Greek letter “mu” and  $\nu$  is the Greek letter “nu.” Those Greek letters are just placeholders for numbers that indicate which number in the matrix we're talking about. So for instance, if  $\mu = 0$  and  $\nu = 1$ , I'm specifically referring to  $g_{01}$ , the top right number in the matrix. So say I had,

$$g_{\mu\nu} = \begin{bmatrix} 4 & 7 \\ 9 & 3 \end{bmatrix}. \quad (10)$$

Then I could say for  $\mu = 0$  and  $\nu = 0$ ,  $g_{\mu\nu} = g_{00} = 4$ . Similarly, for  $\mu = 0$  and  $\nu = 1$ ,  $g_{\mu\nu} = g_{01} = 7$ . Then  $g_{10} = 9$  and  $g_{11} = 3$ . Not so bad right? All the matrix and the Greek letters are doing is providing a convenient way to package together four numbers. I could write down matrices of other shapes too. What about:

$$dx_\mu = \begin{bmatrix} dx_0 \\ dx_1 \end{bmatrix}. \quad (11)$$

For  $\mu = 0$ , we're referring to the number  $dx_0$  and for  $\mu = 1$ , we're referring to the number  $dx_1$ . Note that the Greek letter I'm choosing to use doesn't matter here. I can use  $\lambda$  (lambda) and write  $dx_\lambda$  or  $\gamma$  (gamma) and write  $dx_\gamma$ . The Greek is, for the most part, just a placeholder! It only serves a purpose when we write something like this:

$$g_{\mu\nu} dx_\mu dx_\nu. \quad (12)$$

When you see the Greek letters repeated like this (see how the  $\mu$  appears twice and the  $\nu$  appears twice), we need to do some adding. Specifically, we take all the combinations of  $\mu$  and  $\nu$  (i.e.,  $\mu = 0$  and  $\nu = 0$ ,  $\mu = 0$  and  $\nu = 1$ ,  $\mu = 1$  and  $\nu = 0$ ,  $\mu = 1$  and  $\nu = 1$ ) and add them up:

$$g_{\mu\nu} dx_\mu dx_\nu = g_{00} dx_0 dx_0 + g_{01} dx_0 dx_1 + g_{10} dx_1 dx_0 + g_{11} dx_1 dx_1 \quad (13)$$

But remember that all of these are just numbers, so  $dx_0 dx_0 = dx_0^2$  and  $dx_1 dx_1 = dx_1^2$ , so this becomes,

$$g_{\mu\nu} dx_\mu dx_\nu = g_{00} dx_0^2 + g_{01} dx_0 dx_1 + g_{10} dx_1 dx_0 + g_{11} dx_1^2 \quad (14)$$

Aha! Look!!! Glance back to equation 8 and compare to this. If  $g_{00} = 1$ ,  $g_{01} = 0$ ,  $g_{10} = 0$ , and  $g_{11} = 1$ , and then we let  $dx_0 = dx$  and  $dx_1 = dy$ , these two equations are exactly the same!!! Writing it all out this way, we have

$$ds^2 = dx^2 + dy^2 = g_{\mu\nu} dx_\mu dx_\nu \quad (15)$$

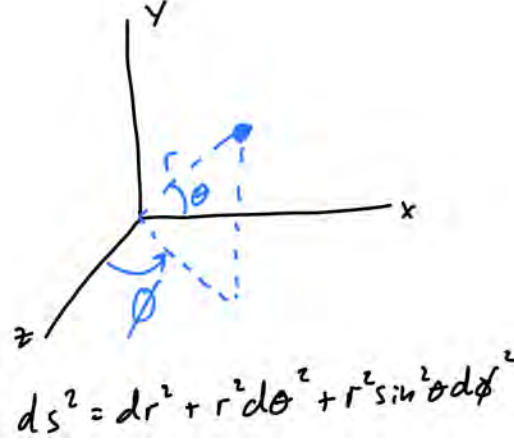


Figure 11

for the specific case that

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (16)$$

and

$$dx_\mu = \begin{bmatrix} dx \\ dy \end{bmatrix}. \quad (17)$$

So where does that leave us? Well, we've basically just written the Pythagorean theorem in a really, really complicated way (namely,  $ds^2 = g_{\mu\nu} dx_\mu dx_\nu$ ). Why would we do such a thing??? Good question. Well first off, this way of writing the Pythagorean theorem is super general. If we know other ways of writing  $ds^2$ , they all look the same when written this way. For instance, remember that we said  $ds^2$  also equals  $dr^2 + r^2 d\theta^2$ . If we let  $g_{00} = 1$ ,  $g_{01} = g_{10} = 0$  and  $g_{11} = r^2$  and then let  $dx_0 = dr$  and  $dx_1 = d\theta$ , we have

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu = \begin{bmatrix} 1 & 0 \\ 0 & r^2 \end{bmatrix} \begin{bmatrix} dr \\ d\theta \end{bmatrix} \begin{bmatrix} dr \\ d\theta \end{bmatrix} = dr^2 + r^2 d\theta^2 \quad (18)$$

If we know what to put in the different matrix slots, we can always write the Pythagorean theorem this way. Another reason for using this complicated looking way of writing  $ds^2$  is that it is very compact. Imagine if we wrote out the Pythagorean theorem in three dimensions like we did earlier:

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = dx^2 + dy^2 + dz^2 \quad (19)$$

The expression  $ds^2 = dx^2 + dy^2 + dz^2$  doesn't look too bad. But imagine if the  $g$  matrix weren't full of zeroes. Or if  $g$  were 4 by 4 instead of 3 by 3 or 2 by 2. There would be a lot of terms in the Pythagorean theorem! And physicists are lazy, so we avoid writing at all costs. Even the expression for the Pythagorean theorem in three



dimensions using a different set of coordinates  $(r, \theta, \phi)$ , shown in Figure 11, is a lot to write:

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2 \theta \end{bmatrix} \begin{bmatrix} dr \\ d\theta \\ d\phi \end{bmatrix} \begin{bmatrix} dr \\ d\theta \\ d\phi \end{bmatrix} = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (20)$$

Ok. We're finally done. We've built up all the notation that we need to gain some insight into what Einstein did in his work. Even if you didn't really get everything that we just did, here are the main takeaways from this section:

- The Pythagorean Theorem is telling you about the distance between two points (the length of the long side of the triangle)
- The distance  $ds^2$  that the Pythagorean theorem tells you about is the same regardless of the coordinates you use to tell where the triangle is (As it should be! It wouldn't make sense otherwise!)
- $g_{\mu\nu}$  is basically a fancy way of writing the Pythagorean theorem. Thus, **the matrix  $g$  encodes information about distances between points**

Lastly, some vocabulary. It turns out that  $g_{\mu\nu}$  is quite important (otherwise I wouldn't have spent so much time talking about it), and has its own name. It's called the **metric tensor**.

## Einstein!

### Einstein's Special Theory of Relativity

Back to the story. When we last left our heroes, there was an epic showdown between the ideas of Galileo and Maxwell over how to add velocities, like whether someone would see a higher speed of light if you drove by in a car and turned on a flashlight. In 1905 Einstein was quite concerned with this debate (among other things - Einstein did a lot of physics). One of the things that Einstein figured out in his **Special Theory of Relativity** was that *Maxwell* was right, not Galileo. The speed of light is constant no matter what. The person on the side of the road measures the speed of light as  $c$  and the person in the car measures the speed of light as  $c$ . No matter what,  $c = c$ . So what are the consequences of Einstein's assertion that the speed of light be constant for all observers?

Perhaps the quickest way to see one of the most significant consequences of a constant speed of light is through a thought experiment that Einstein himself came up with. Imagine that you want to build a clock using light. How might you do that? Well, you could put a device together like in Figure 12a). What that image shows is two mirrors (the black lines), separated by a distance  $L$ . If you shined a beam of light up from the bottom mirror (shown in blue), the light would go up, bounce off the

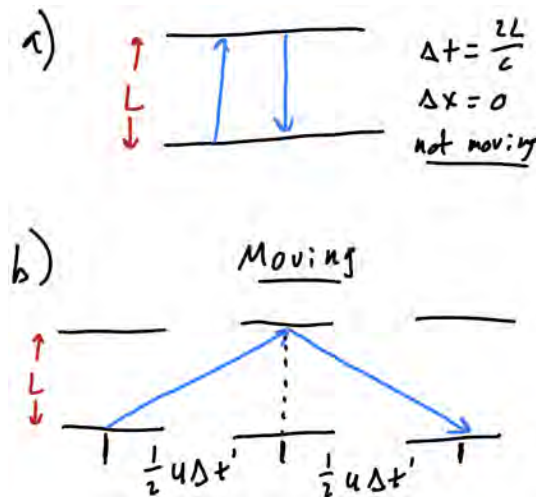


Figure 12

top mirror, and come down. Since we know the speed of light is  $c$ , and the distance it traverses in one up-and-down motion is  $2L$ , then the time the light took to go up and down must be  $\Delta t = 2L/c$ . This is just reorganizing the speed equals distance over time equation that we learned on the first day of class. So someone sitting there looking at this fancy clock knows how much time has passed for each “tick” (up-and-down motion) of the light ( $\Delta t = 2L/c$ )!

Now consider the following. The image in Figure 12a) is what the clock would look like for someone sitting still next to the clock. But what if the person picked up the clock and started running down the street with it? Well, that person is moving along with the clock, so it still looks the same. For the running clock carrier, each tick still takes  $\Delta t = 2L/c$ . However, things look a bit different for someone sitting off to the side watching the clock move by. This is shown in Figure 12b). The light still goes up and down, but as it’s doing so, the whole clock is moving to the right. Thus, it looks like the light has to travel at an angle to stay moving with the clock.

For Galileo, this isn’t a big deal. But for Einstein, this is a *huge* deal!!! Why? Because for the person watching the clock move by, *they think the light has to travel a larger distance in one tick than does the person running along with the clock!* But if the speed of light is the same for everyone, that means the light will have to traverse a longer distance, and one tick will therefore take *more time!*

So what does this mean? For a person sitting still, they think that the person running has a clock that is ticking too slowly. To be clear: this isn’t just an illusion or a trick. One immediate prediction of Einstein’s special relativity is that if you are moving, time moves more slowly for you. This is called **time dilation** and it is a very real phenomenon. We’ve verified it experimentally over and over! In one rather famous test, physicists synchronized two clocks and sent one up in an airplane to fly around the Earth for awhile. Sure enough, the one moving started to lag behind the one

sitting on the surface of the Earth. However, this effect is very small unless you're moving very, very fast. The difference in the times between the clock in the plane and on the Earth was only a couple hundred nanoseconds. To see major time dilation effects, one would need to be traveling close to the speed of light. We don't ever move that fast, but many small particles reach those speeds routinely. In those cases, we observe that particles that should decay quickly actually "live" longer than they should because time is moving more slowly for them! In addition to the time dilation effect, special relativity also predicts **length contraction**, where the moving person thinks things are shorter than they really are. These two effects already show that some weird stuff is going on due to Einstein's idea that the speed of light is constant for all observers!

Now, since we spent so much time talking about the Pythagorean theorem and geometry, we're actually only a hop skip and a jump away from connecting to Einstein's *general* theory of relativity, which is where he really did some crazy physics.

Let's go back to that clock for a moment and do some math. Suppose that the person running with the clock is running with a speed  $u$ . And since we already know that the person watching the clock go by is going to measure a different tick time than the person running with the clock, let's label the person watching the clock's tick by  $\Delta t'$  to differentiate it from  $\Delta t$ , the tick of the person running with the clock. Looking back at Figure 12b), we can use the Pythagorean Theorem to write down the distance traveled by the light in one tick. The bottom of the triangle is half the distance traveled by the running person, namely  $u\Delta t'/2$ . It will be useful to simply label this distance  $\Delta x'/2$  - the distance they ran to the right according to the person sitting off to the side watching them run. The height of the triangle is just the distance between the mirrors,  $L$ . And since the light travels at speed  $c$ , the hypotenuse (the distance the light travels) is given by multiplying  $c$  by  $\Delta t'$  and dividing by 2 (since it's only half the tick - the "up" part). Putting this all into the Pythagorean theorem, we have the equality

$$c\Delta t' = 2\sqrt{(\Delta x'/2)^2 + L^2} \quad (21)$$

Squaring both sides and subtracting the  $(\Delta x')^2$  term from both sides gives

$$(c\Delta t')^2 - (\Delta x')^2 = 4L^2. \quad (22)$$

But wait! We saw earlier that for the person running alongside the clock,  $\Delta t = 2L/c$ . Thus,  $4L^2 = (c\Delta t)^2$ , meaning that

$$(c\Delta t')^2 - (\Delta x')^2 = (c\Delta t)^2. \quad (23)$$

This is starting to look familiar... Let's do one more thing. It may seem weird to do, but it is completely legal to add zero to a number.  $2 + 0$  is still equal to 2. So if I add or subtract a bunch of stuff to both sides of the previous equation that's all equal to zero, then both sides are still equivalent. Moreover, note that for the person

running alongside the clock, the clock isn't moving, so for that person the change in the clock's position  $\Delta x$  is zero. Similarly, that person doesn't think the clock moved up or down or side to side, so  $\Delta y = \Delta z = 0$  too. Meanwhile, we already said that for the person on the side of the road watching the clock go by, the clock's position changes by  $\Delta x' = u\Delta t'$ , but it doesn't move in either of the other directions. So  $\Delta y' = \Delta z' = 0$ . Thus, we can add or subtract all the stuff equal to zero and get:

$$(c\Delta t')^2 - (\Delta x')^2 - (\Delta y')^2 - (\Delta z')^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2. \quad (24)$$

What does this mean? Well, it means that the expressions  $(c\Delta t')^2 - (\Delta x')^2 - (\Delta y')^2 - (\Delta z')^2$  and  $(c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$  are the same for all observers moving relative to each other. We call quantities like this *invariant*. So the quantity  $(c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$  is the same for all observers, and it looks really familiar - very much like all the geometric expressions and Pythagorean theorem things we looked at earlier. In fact, they are deeply related. Hermann Minkowski asserted that

$$ds^2 = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2 \quad (25)$$

is the distance (just like Pythagorean theorem!) between *spacetime* points. Not just space, but spacetime - Minkowski put time and space on equal footing. And just like how changing coordinates in the Pythagorean theorem didn't change  $ds^2$ , the relative motion between observers doesn't change  $ds^2$  here. This insight was the basis of being able to describe space and time together as one.

Since this expression looks a lot like the ones we saw earlier, let's write

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} cdt \\ dx \\ dy \\ dz \end{bmatrix} \begin{bmatrix} cdt \\ dx \\ dy \\ dz \end{bmatrix} = (cdt)^2 - dx^2 - dy^2 - dz^2 \quad (26)$$

This particular form of the metric tensor  $g_{\mu\nu}$  is special because it describes our local spacetime, and so it gets a special name: the Minkowski Metric.

## Einstein's equivalence

Now for one last topic before we talk about the General Theory of Relativity: Einstein's equivalence principle. Einstein's equivalence principle basically says that you can't tell the difference between the two scenarios shown in Figure 13. On the left, someone is sitting in a spaceship that is stationary on Earth and drops a ball. They see it fall with acceleration equal to  $g$ , the acceleration due to gravity (not like the  $g_{\mu\nu}$  matrix from earlier). On the right, someone standing in a spaceship that is in deep space accelerating *up* at  $g$  and drops a ball. The ball still looks like it moves down with acceleration  $g$  to the person in the spaceship! Similarly, if you're in free-fall, you can't tell if you're in a region of no gravity or not. Astronauts train for zero-gravity

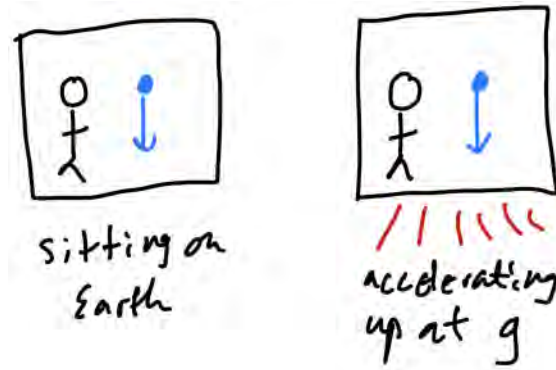


Figure 13

missions on the moon by flying in an airplane and letting it fall down back to Earth - freely falling like this feels the same to them as when they're out in space with no gravity. So free falling in gravity “feels” like there’s no force on you at all.

Einstein took these simple but profound observations and combined them with a bit of Newton’s insight from hundreds of years earlier. Remember Newton’s First Law says that if you’re not experiencing a force, then you’re moving at a constant velocity in a straight line. Since falling in gravity “feels” like there’s no force on you, Einstein essentially extended this to posit that moving in gravity is the same as moving on a “straight line,” but in a curved spacetime (such a line is called a *geodesic*). And he was right.

What does this look like? Perhaps the best visual is to think of a bowling ball on a trampoline. The bowling ball makes the fabric around it curve, and if you were to drop a marble on the outside of the trampoline, it would travel toward bowling ball. If you gave it some speed, it would orbit around the bowling ball before falling in. If you gave it just the right speed, it would orbit for quite awhile. This is just like the moon and the Earth, or the sun and the planets. The planets are merely traveling on “straight lines” in the spacetime that has been curved by the sun’s mass. *This was the mechanism for gravity that Newton wanted to understand hundreds of years before!*

## Einstein’s General Theory of Relativity

So Einstein had the right idea. But he needed to come up with a brand new way of thinking about how gravity works based on the curvature of spacetime. And that’s hard. Because you’re talking about curved spacetime, you’re talking about doing geometry in 4 dimensions where stuff is... well... curvy, and things generally don’t work the same as you’d expect.

**But at the core of it all is the metric tensor!!!.** In fact, the final result of working on General Relativity for 10 years (his special theory of relativity came out

in 1905, the general theory came in 1915) was the **Einstein tensor**,  $G_{\mu\nu}$ . He found a set of equations, called Einstein's Equations, given by

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (27)$$

where  $G$  is regular old Newton's gravitational constant and  $T_{\mu\nu}$  is called the stress-energy tensor, which describes the amount of mass and energy. So what this equation says is that curvature (the Einstein tensor) is equal to the stuff in the spacetime (the stress-energy tensor). Gravity is geometry!

As it turns out, the Einstein tensor is a (very messy) function of the metric tensor! This is perhaps unsurprising, since the Einstein tensor describes the curvature of spacetime, and the metric is all about encoding distances in spacetime. So how does one calculate the Einstein tensor? Well, that's a little beyond the scope of these notes. That would require many more pages. But I can write out the steps to show you how the metric tensor plays such a central role, and to give you an idea of how in-depth the overall process is and why it took Einstein 10 years to figure it out.

Given a metric tensor  $g_{\mu\nu}$ ...

1. Calculate the **Christoffel Symbols**:

$$\Gamma_{\mu\nu}^{\rho} = \frac{1}{2} g^{\rho\lambda} (\partial_{\mu} g_{\nu\lambda} + \partial_{\nu} g_{\mu\lambda} - \partial_{\lambda} g_{\mu\nu})$$

2. Calculate the **Riemann Curvature Tensor**:

$$R_{\rho\mu\nu}^{\sigma} = (\partial_{\mu} \Gamma_{\nu\rho}^{\sigma} + \Gamma_{\mu\kappa}^{\sigma} \Gamma_{\nu\rho}^{\kappa}) - (\partial_{\nu} \Gamma_{\mu\rho}^{\sigma} + \Gamma_{\nu\kappa}^{\sigma} \Gamma_{\mu\rho}^{\kappa})$$

3. Then the **Ricci tensor**:

$$R_{\mu\nu} = R_{\mu\sigma\nu}^{\sigma} = g^{\tau\rho} R_{\tau\mu\rho\nu}$$

4. And the **Ricci scalar**:

$$R = g^{\mu\nu} R_{\mu\nu}$$

5. From which we (finally) get the Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

So we went all the way from the metric tensor  $g_{\mu\nu}$  all the way up through the Einstein tensor,  $G_{\mu\nu}$ . See all the places that the metric shows up in these equations? It's important! And at its core it's really just kind of like the Pythagorean Theorem that you learned so long ago! Now, if  $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ , then we've got a solution to Einstein's equations and we can describe how stuff moves in that spacetime around a particular mass!

But what solutions are there? This was actually a source of disappointment for Einstein. He figured that his equations were so complex that a solution would be basically impossible to find. Enter an individual with one of the most interesting backstories in the history of physics, Karl Schwarzschild. Schwarzschild was a German soldier in World War I who also happened to do physics. In 1916, mere months after Einstein first published his General Theory of Relativity, Schwarzschild was laid up in the hospital after being injured in the war. While there, he figured out the first solution to Einstein's equations. Yes, a WWI soldier in the hospital solved Einstein's Equations. Schwarzschild wrote a letter to Einstein that said, "As you can see, the war treated me kindly enough, in spite of the heavy gunfire, to get away from it all and take a walk in the land of your ideas." Wow.

So what was Schwarzschild's solution? He found that the metric

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \frac{1}{(1 - 2GM/r)}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \quad (28)$$

satisfied Einstein's equations. Don't worry about the details, I'll point out some of the important parts. This metric was supposed to describe the gravity outside a spherically symmetric body of mass  $M$  such as a star or planet. And it indeed does that. In fact, it fixed the discrepancy in predictions of Mercury's perihelion shift, one of the first confirmations that Einstein's new theory was correct.

Another consequence of Einstein's gravity is that since gravity is a curvature of space-time, it also affects light even though light is massless. Einstein predicts that light also travels along in curved spacetime and gets affected by a massive object's gravity. This is actually one of the methods that we use to detect exoplanets. When they pass in front of stars, even if we can't see the exoplanet, we can see that it's bending the star's light. This effect is called Gravitational Lensing.

Looking at the metric, note that it appears something weird happens at  $r = 2GM$  ( $r$  is the distance from the spherically symmetric body). When  $r = 2GM$ , the bottom of the fraction in front of the  $dr^2$  goes to zero. But that seems like a problem, since dividing by zero is impossible! So what's going on there? Turns out that value is the *event horizon* of a *black hole*. Schwarzschild's metric implies the existence of black holes, which are extremely massive objects that have gravity that is strong enough to pull in light. The event horizon is the radius at which light will always travel to the center of the black hole - the light can't move fast enough to escape its pull. You might notice that at  $r = 0$  there are also several fractions that are divided by zero in Schwarzschild's metric. This is a genuine singularity, and we don't know exactly what's going on here. We need more sophisticated physics that has yet to be invented in order to say more.

Another effect predicted by Einstein's gravity is the gravitational redshift. Just as SR predicted that moving fast would slow down time around you, GR predicts that being in the presence of a massive body also slows down time. If you've ever seen

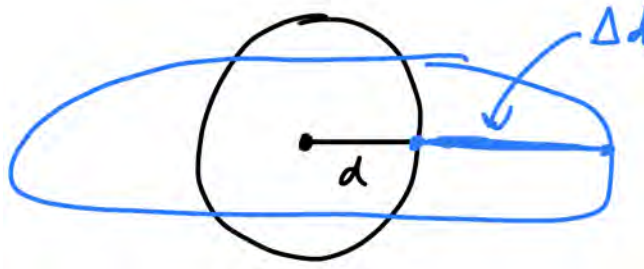


Figure 14

the movie *Interstellar* with Matthew McConaughey, there's a scene where they end up on a new planet and someone tells him that an hour on that planet is equivalent to years on Earth because of the planet's gravity. That's a real effect (assuming one could get to such a planet)!

Lastly, I'll mention one of the most recent verifications of Einstein's General Theory of Relativity, the detection of *gravitational waves*. Einstein's theory predicts that massive objects like black holes moving through spacetime cause ripples in it, kind of like a boat through water. These waves are *super* small, and travel at the speed of light. Their amplitudes are measured in "strain" (depicted in Figure 14), which is defined as the fractional change in displacement between two nearby masses due to a gravitational wave (see in the image how the blue mass shifts away from the black mass in the middle). The strain depicted in the image would be equal to  $\Delta d/d$ . For context, the strain of the gravitational waves detected several years ago by LIGO (read about LIGO in your workbook!) was about  $10^{-20}$ , which is incredibly small. And those waves were due to the collision of two black holes!



# A very brief slice of Quantum Mechanics (1)

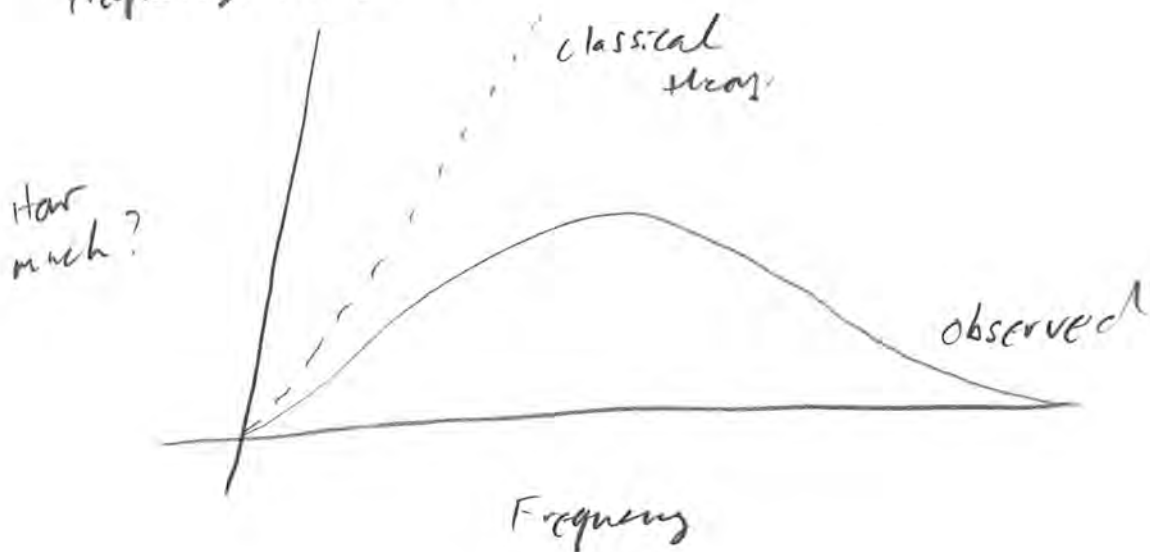
throughout class we've taken for granted that things are made of atoms, molecules, etc. and that protons, electrons, neutrons etc. exist. the world looks continuous!

that wasn't always obvious! And in fact around  $\approx 1850-1900$  people (Faraday, Maxwell, Hertz, many others) were doing experiments and theory that started to contradict "classical" physics (classical meaning not quantum or Einstein).

## Famous examples:

Black body radiation - remember, things with temperature emit radiation. People (Gustav Kirchhoff) circa 1860 ish noted something wrong.

Rayleigh - Jeans said that classically, the light emitted by an object should get more and more intense as frequency increased.



This was called (famously) the "ultraviolet catastrophe" ②  
- Resolved Dec. 14 1900 by Max Planck in  
"On the theory of the Energy Distribution Law of  
the Normal Spectrum"

"the Birthday of Quantum Physics"

Resolved by saying that the energy levels of  
light could only take on discrete values -

very much not classical -  $E = h\nu$

$h$  = Planck's constant =  $6.63 \times 10^{-34}$  joule-sec

## Photoelectric Effect:

1887 Hertz finds that shining light  
on metal makes electrons in the metal  
become "detached" from it - called the photoelectric  
effect.

↑ basically  
how solar  
cells work

Classically - more intense light means more  
energy so any light, if intense  
enough, should be able to do this.

But this wasn't the case! Each metal had a frequency of  
light below which no electrons would come out.

$$E = h\nu$$

Einstein - 1905 - light is "quantized" into  
photons  $\rightarrow e^-$  "wave-particle  
duality"

the continuous world is looking more and more discrete! (3)

How do these reconcile?

- when you have lots of photons, the "continuous" waves are a kind of average property of all the photons
- Kind of like a garden hose - the stream of water has so many individual droplets you don't see the individual behavior, just the whole stream.

Starts to introduce the idea that some of this quantum stuff is related to averages, statistics, probability

which is where weird stuff happens.

Enter LOTS of people -

Max Born <sup>collaborated with UR folks!</sup>, Schrödinger, Bohr, Heisenberg,

de Broglie, Einstein, Dirac, Pauli, etc etc

1927 Heisenberg and Bohr argue that probability is actually quite fundamental to nature.

Heisenberg Uncertainty Principle:

$$\Delta p \Delta x \geq \hbar/2 \quad (\hbar = \text{"h bar"} = \frac{h}{2\pi})$$

We can't simultaneously know the position and momentum of matter at the same time!!

1925 Schrödinger tries to reconcile the whole wave/particle thing for microscopic particles that clearly show wave properties.

Schrödinger's Equation:

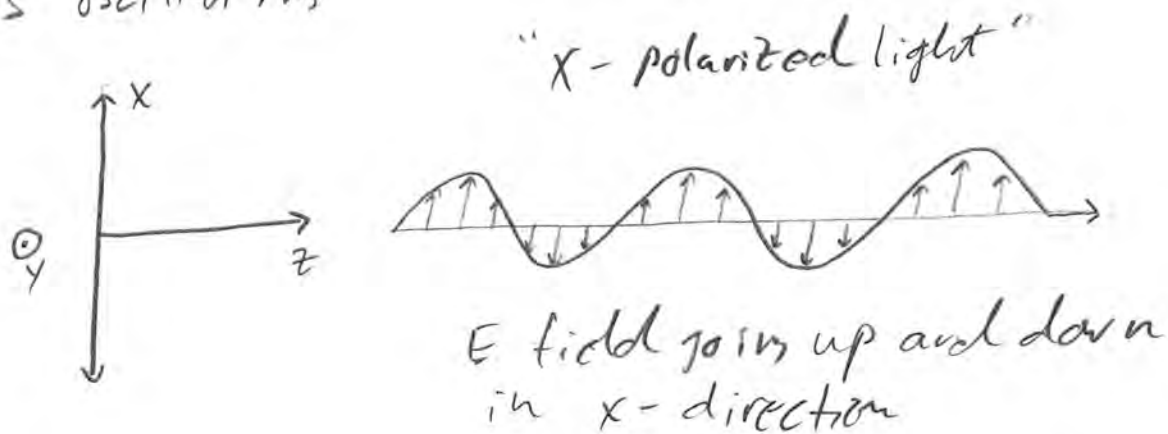
$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

$\psi$  is a "wave function" that contains information about where a particle is most likely to be, not where it definitely is!

# A small digression on Light Polarization

(5)

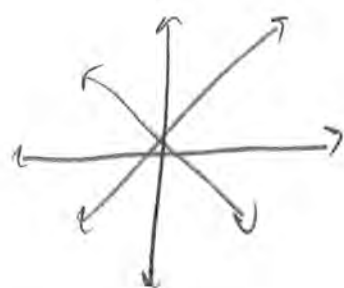
A light wave propagating along can be polarized, which refers to the direction that the wave's electric field is oscillating



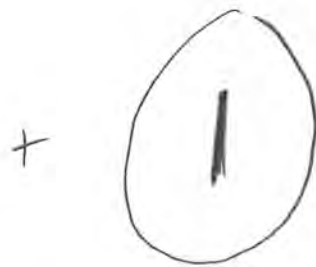
can have y-polarized, z polarized, etc.

How to get?

Send light beams that are unpolarized (the sun, light bulbs) through a polaroid Filter



E fields



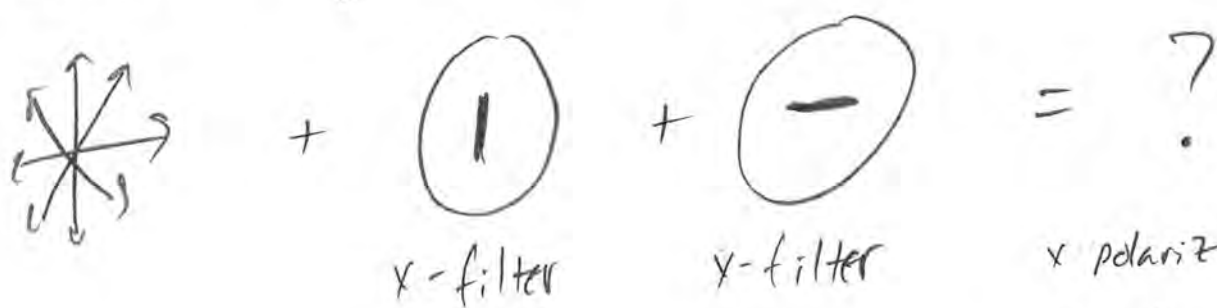
x filter



x-polarized light

So what happens here?

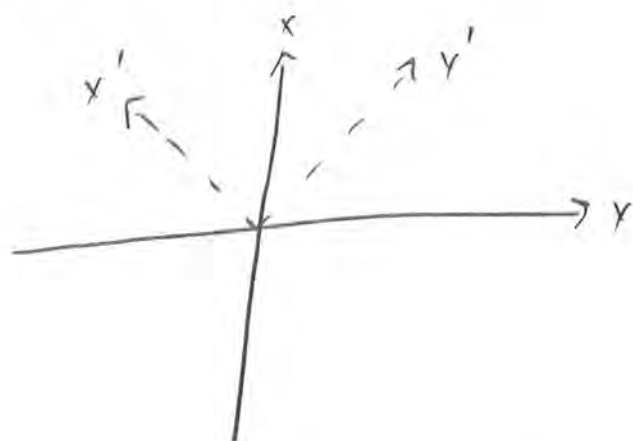
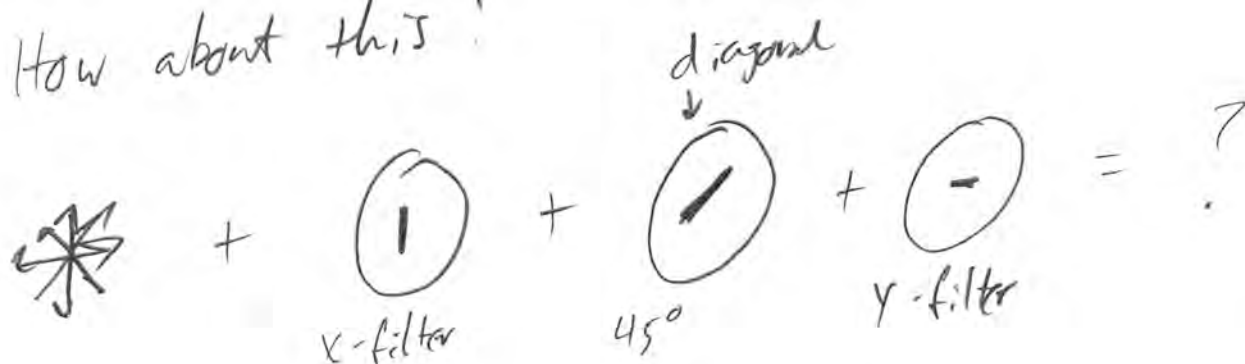
(6)



$$\begin{aligned}x \text{ polarized} &= |x\rangle \\y \text{ polarized} &= |y\rangle \\ \langle y|x\rangle &= 0\end{aligned}$$

No light!

How about this:



$$|x'\rangle = \frac{1}{\sqrt{2}} |x\rangle - \frac{1}{\sqrt{2}} |y\rangle$$

$$|y'\rangle = \frac{1}{\sqrt{2}} |x\rangle + \frac{1}{\sqrt{2}} |y\rangle$$

and

$$|x\rangle = \frac{1}{\sqrt{2}} |x'\rangle + \frac{1}{\sqrt{2}} |y'\rangle$$

$$|y\rangle = -\frac{1}{\sqrt{2}} |x'\rangle + \frac{1}{\sqrt{2}} |y'\rangle$$

Why write in such a weird way?

(7)

"multiply" by  $|y\rangle$  (ie, measure how much is in  $y$  direction)

$$\langle y | x' \rangle = \frac{1}{\sqrt{2}} \cancel{\langle y | x \rangle} + \frac{1}{\sqrt{2}} \langle y | y \rangle$$

$$\langle y | x' \rangle = \frac{1}{\sqrt{2}}$$

Square it, get amount of light  $\frac{1}{2}$

Triple filter!  $x \rightarrow x'$   
 $x' \rightarrow y$

$$\langle x' | x \rangle = \frac{1}{\sqrt{2}} \quad \text{Squared } \frac{1}{2}$$

$$\langle y | x' \rangle = \frac{1}{\sqrt{2}} \quad \text{Squared } \frac{1}{2}$$

So each time,  $\frac{1}{2}$  gets through. In total  
 $\frac{1}{4}$  gets through!

Fore shadow: what if the light is an individual  
photon, so you can't speak about "intensity" anymore?

Probability!

- Turns out, this is a direct analogy with wave functions, <sup>(8)</sup> where intensity gets swapped with probability
- Say a particle can be in states  $x, y, x', y'$  related by the same equations. If the particle starts in state  $|x\rangle$ , there's a  $1/2$  chance of measuring it in state  $|x'\rangle$ . And there's a  $1/4$  chance that measuring it in the  $|y\rangle$  state after you measured whether it was in the  $|x'\rangle$  state.

Bring back to Schrodinger etc.

A particle has wavefunction

$$|\psi\rangle = \frac{1}{\sqrt{2}}|x\rangle + \frac{1}{\sqrt{2}}|y\rangle$$

What is the probability of finding the particle in state  $x$ ?

$$|\langle x|\psi\rangle|^2 = \left(\frac{1}{2}\right)$$

Why is this interesting?

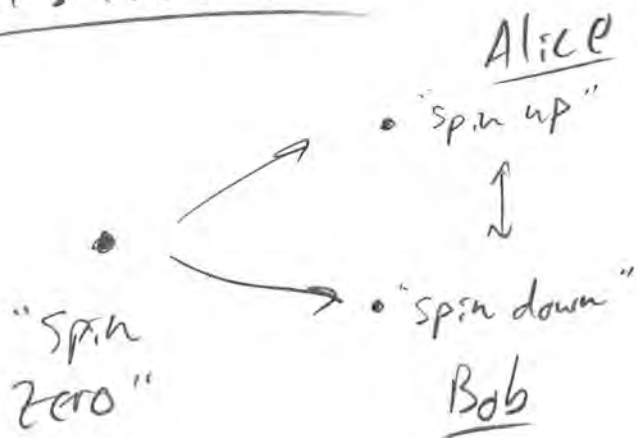


The equation  $|4\rangle = \frac{1}{\sqrt{2}} |x\rangle + \frac{1}{\sqrt{2}} |y\rangle$  means that (9)  
the particle is in both states at once - a  
superposition of states. (Kind of like interfering waves)

You "collapse the wavefunction" when you make  
a measurement.

this is weird, and it really isn't an either/or  
thing. The state isn't predetermined - this bothered  
Einstein a lot!

### Einstein's problem



Spin zero decays. To  
conserve angular momentum,  
the two new particles  
have total angular momentum  
zero, but we don't know  
which is which.

Particles are described by wave functions like

$$|4\rangle = \frac{1}{\sqrt{2}} |\uparrow\downarrow\rangle - \frac{1}{\sqrt{2}} |\downarrow\uparrow\rangle$$

particle<sub>1</sub>      particle<sub>2</sub>

50% chance Alice gets  $\uparrow$  Bob gets  $\downarrow$   
50% chance Alice gets  $\downarrow$  Bob gets  $\uparrow$

(10)

These results are hooked together though - once Alice measures hers, Bob's is instantly determined!

So you could separate Alice and Bob by light years and one measurement determines both results simultaneously. Seems like faster than light!

"spooky action at a distance"

"God does not play dice with the universe"

"Do you really believe the moon isn't there unless you look at it?"

John Bell  
Bell's Inequality

## Quantum Entanglement

Schrodinger knew in  $\sim 1935$ , not realized it could be useful until  $\sim 1990$ 's.

Closest thing to teleportation that we have :)  
(see next page)

## Transfers information

Some sources of trouble - decoherence... can't let the entangled things get disturbed or they won't be entangled anymore!

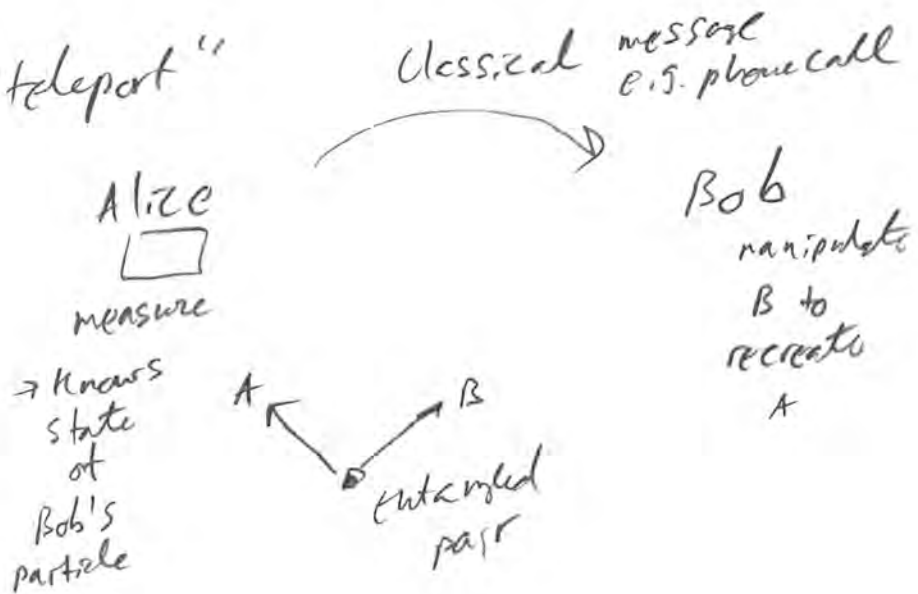
Spin coherence circa 2008  $\sim$  picoseconds ( $\times 10^{-12}$ )

Nature 2011 reported  $\sim$  ms ( $\times 10^{-3}$ )

Some experiments have teleported information with light  
- entangled photons in fiber optic

(11)

How to "teleport"



## The Second Quantum Revolution

Harnessing the power of quantum. Quantum computing,  
Quantum sensing, Quantum cryptography. Information

### Quantum Computers

Classical computers first

Basically ----  
Computers store and process information as bits - 1's and 0's

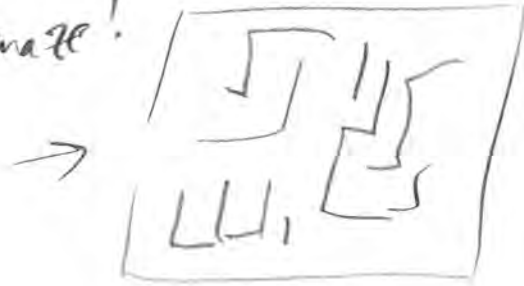
When you save a number on a computer, it's stored  
as bits that the computer knows how to interpret

e.g. the number 15 in "binary" is 1111

5 is 101

Classical computers do calculations by trial and error (12)

Solve a maze!



Classical computer tries each path one by one until it gets the right answer

Why is quantum different?

Remember, quantum particles (call them "qubits" in a quantum computer) exist in multiple states at once

$$|4\rangle = \alpha |0\rangle + \beta |1\rangle$$

Both zero and one at once. one bit = 2 values  
one qubit = 2 values

2 bits

0	0	0
0	1	1
1	0	2
1	1	3

2 qubits

$$|4\rangle = \alpha_1 |00\rangle + \alpha_2 |01\rangle + \alpha_3 |10\rangle + \alpha_4 |11\rangle$$

2 qubits take on all the values of the 4 bit combinations at once stores all the numbers at once

3 bits

0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

3 qubits take on all the values of the 8 bit combinations at once.

Qubits pack more information

This scales quickly

(13)

$$13 \text{ qubits} \rightarrow 8192 \text{ bits} \rightarrow 1 \text{ kb}$$

$$43 \text{ qubits} \rightarrow 8.8 \times 10^{12} \text{ bits} \rightarrow 1 \text{ TB}$$

$$63 \text{ qubits} \rightarrow 9.2 \times 10^{18} \text{ bits} \rightarrow \text{Exabyte}$$

How does this translate to the maze?

- Quantum computer takes each path at once...  
Measurements on one qubit influence them all
- Take all paths of the maze at once is way faster than doing them over and over

Practical application - Security

How are important numbers stored?

prime numbers

2 3 5 7 11 13 ...

$$3 \times 5 = 15$$

$$7 \times 5 = 35$$

> only divisible by the two prime numbers!

$$17 \times 19 \times 23 = 7,429 \text{ easy!}$$

$$7,429 = ?? \text{ hard!}$$

this is the heart of RSA data encryption (14)  
Take two huge <sup>100's of digits</sup> prime numbers and multiply them together. Get an answer.

If you only knew the answer, it would be virtually unsolvable to know what the 2 prime numbers ("keys") are that made it.

Would take hundreds of years for a regular computer to figure it out

Peter Shor 1985 - Shor's algorithm  
• Shows a way to factor prime numbers of any size, but with a quantum computer

• Can see why - could test lots of combinations at once! Takes moments, not centuries

Big deal - a quantum computer that could execute this consistently could break most popular encryption methods in a few moments!

2012 - 21  
2019 - 35  
failed

I B M - 2001 7 qubits, showed Shor's algorithm works by factoring 15 into  $3 \times 5$

Why difficult? Decoherence, breaking entanglement  
Active area of research